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Grothendieck, the harvest
Olivia Caramello, Alain Connes, Laurent Lafforgue interviewed by Nicolas Martin

Nicolas Martin : To say of *R´ecoltes et semailles* that it is the work of a lifetime, is to say everything and say nothing at all at the same time. For a long time, this work in the form of a typescript of more than 1000 pages was surrounded by a form of legend, passing from hand to hand more or less clandestinely, the story telling of elsewhere that it was my esteemed colleague and predecessor St´ephane Deligeorges who was entrusted with the first definitive copy by Grothendieck himself to try to find him a publisher. Alas, this sum which borrows as much from mathematics as from literature, from philosophy, from mysticism, from politics or from ecology, this sum was confined for a long time to to be accessible only via an online digital file, until this month of January: the publisher Gallimard, with the help of IHES, has therefore *published Récoltes et semailles,* in a box of two volumes from the Tel collection, and to enter into the iconoclastic thought of Alexandre Grothendieck, I simply suggest that you listen to him, in the way he approaching mathematics education, we are at CERN in 1972.

Alexander Grothendieck *(recorded archive) :* I am going to be confronted this fall with this situation for the first time in my life by the way, of being in an amphitheater with students to whom I must for good teach the mathematics which will prepare them for certain examinations, providing them with certain diplomas, which I am for my part convinced are knowledge which is useless: on the one hand which are useless for society as a whole but on the other hand, which it is not even clear that they are of any use for those who are going to have this diploma, because it does not It is absolutely not clear that this will allow them to have a job afterwards. So what most scientists still do is either refuse to see the problem or, if they see it, put a public veil over it in their dealings with students. The relationships between the students and themselves are therefore traditional teacher-student relationships; that is to say, they do a technical course, the one we ask of them, that's all. When, exceptionally, students ask technical questions, we answer these technical questions as best we can. As far as I am concerned, I have decided not to limit myself to this type of relationship and no longer to separate the teaching of mathematics from an open-faced discussion with students or anyone who wants to come and attend the discussion to try to make the point: “Why are we here?” ; “What are we going to learn together?” ; “What does the exam at the end of this year's program mean?” ; "What is its meaning ?" ; “What is our mutual role, me teacher and you students?”. And decide together on what we will do.

Nicolas Martin : Here we are, Alexandre Grothendieck in 1972, so 2 years after he left IHES.
Laurent Lafforgue, a reaction to what you have just heard.

Laurent Lafforgue : Yes, this passage is extremely interesting and it immediately allows us to realize two essential features of Grothendieck's thought. On the one hand, the fact that he

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Program The scientific method can be heard here: https://www.franceculture.fr/emissions/la-methode scientist/grothendieck-la-moisson

Transcription Denise Vella-Chemla, February 2022.

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does not separate things, that is to say that he is not on the one hand a mathematician, and on the other hand a person, a writer, he is all that at the same time . We can even introduce Grothendieck's mathematical work by saying that in fact he is not really a mathematician, he is a writer who does mathematics. A second characteristic that appears in this extract is that we hear him ask a fundamental question: ”Why teach mathematics? Why do mathematics? And in fact, this is also characteristic of everything he has done in his life, that is to say, to get to the root of things, and to ask the most fundamental questions, to question all the evidence, that's it. So that's what he did in all of his work and that appears here right away.

Nicolas Martin : He had a certain taste for the letter, Alain Connes, since we hear him say with a certain provocation “because in any case, what I am teaching you will be absolutely useless” and we see that he immediately links, what he will do elsewhere in his work, mathematical education, as Laurent Lafforgue has just said, to everything else, that is to say to the discussion in a much broader field of exchanges with the students and with his public.

Alain Connes : Yes in fact, ¸that, you have to, I mean, if you present ¸that from the outset, well, as Laurent explained it, you can actually understand certain facets of his character. but it still corresponds to a very very specific period in his life and we cannot, I mean, we must not confuse the first impression that the we would have from this period with the evolution of Alexandre Grothendieck. So I will try, on the contrary, if you want, to place myself in the history of its own evolution, as a complement to what Laurent said. And in the story of its own evolution, I want to do that, because I would like to contrast two periods, if you will. Grothendieck had an extremely lonely childhood. He wasn't abandoned but I mean, he stayed away from both his parents from the age of 6, and he suffered a lot from it. But in *Récoltes et semailles,* he clearly explains that solitude is his favorite companion and it is his companion that allows him to be creative, more than anything else. First thing.

Then, if you like, when Grothendieck, through rather harsh vicissitudes in his life, studied at the faculty of Montpellier. When he was 20, he moved to Paris. And there, he was admirably well received by the mathematicians who were Henri Cartan, Dieudonné, Schwartz, Serre, finally therefore, if you like, there was a period blessed.

Nicolas Martin : The founders of IHES moreover...

Alain Connes : No, no, not yet. It is the period in which he began with a subject which is, as he says, rather off-putting, which is functional analysis, in which he found, if you will, for relax the atmosphere a little, he found a notion which is what we call nuclear spaces . So you have to know that this has nothing to do with nuclear physics, although you can disintegrate atomic measurements on nuclear space. So actually, if you like, Grothendieck made a terrific find, he dazzled Dieudonné and Schwartz at that point and then he branched off into what's called geom algebraic ether. And here again, it was admirably well received, if you will, by Jean-Pierre Serre. There is a correspondence between Grothendieck and Jean-Pierre Serre, which is a very large volume, and in which we see how, if you will, these two truly complementary characters made an exchange which been amazing

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ably productive. And afterwards, if you like, thanks to Serre, thanks also to the incredible devotion of Jean Dieudonné, Grothendieck was able to give his full potential in algebraic geometry. As he says in *Récoltes et semailles,* he never struggled, that is to say that for him, it did not represent an effort, he bent down and he reaped everything. what he could, and it was a period, well, he had a difficult passage when his mother died, he had a kind of depression for 6 months, then after es, he had a period of absolutely radiant creativity, in which he invented the notion of topos which we will talk about later, but in 70 he had an extremely abrupt change, that it is very difficult to analyze correctly, there are all sorts of possible analyses, and there is a drawing that he made, which is absolutely striking, in which he wrote witches in German eras, and below this drawing, there is a text in German, in which he explains that he is cooking up one of his main theorems, which is what is called the th Riemann-Roch-Grothendieck aeorem, there is a fire which is fanned by imps ins, and what Grothendieck writes in German below is that to explain this theorem takes 500 pages, and he does not understand that the human mind is devoted to ¸ca, while he feels in his body that life is threatened. And basically, if you will, he makes a profession of ecological faith at that time, he is also frightened of course by the nuclear threat, and he makes a profession of faith that would not be denied. nobody now, 51 years later.

Nicolas Martin : We will obviously discuss all that. I would like to specify two things right away, it is that we had therefore already devoted a program to Alexandre Grothendieck, we are not going to give all his biography, I refer you to on this show if you want to know more, really, about the beginning of his career and all this period that Alain Connes has just summed up brilliantly, we are really going to focus on *Harvests and sowing .*

Olivia Caramello, perhaps a word, already, on this archive that you have heard, and to hear Grothendieck's voice again, with this little accent so significant, and then, more generally , on the link you have to *Récoltes et semailles ;* it was available online, free of charge, it was important to give it body, in a volume, accessible to everyone in paper according to you?

Olivia Caramello : Yes, yes, absolutely, so yes, it's always moving to listen to Grothendieck's voice, so pure and attentive to the precision of expression too. He was, all the same, I want to underline it, very rigorous in mathematics, but he is also so in his anthropological, psychoanalytical, and other kinds of reflection, which is found in great abundance in this marvelous text *Harvests and sowing.*

So yes, I would just like to add a remark regarding what Alain Connes was saying. Indeed, this excerpt from the speech may make him appear as perhaps a little too provocative, that is to say ¸it highlights this side of provocation and ¸it may perhaps if c is not really put in the right context, it can make you think that he really wanted to stop scientific research, and many people interpreted it like that, whereas if you look all the same his scientific production after that period, we can still see that he never stopped doing research, whether in mathematics or in other subjects. So ¸that, I wanted to specify it all the same and Grothendieck, he is really a thinker on a large scale, so he is someone who of course has provided spectacular proof of his talent, especially in maths. ematics, but not only. And in fact, we begin to discover it with in particular this text *Récoltes et semailles*

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which is filled with very, very deep reflections on a whole host of subjects, and also on the whole ethics of mathematics and more generally of scientific activity. So personally, I see this speech at CERN as perhaps an important step in this work of broader reflection, beyond mathematics, which later deepened in the years following ones, and that's it, so, personally, I think it's very very good that *Récoltes et semailles* is finally out. I already knew it, of course through the electronic version, and in fact, for me, it was wonderful to discover this text because it explains a vision , really. It is a text in which Grothendieck takes the time to explain his vision, his vision of mathematics, and also, of course, the relationship he has with the mathematical community, because, in particular , of this vision, of how this vision is received by his contemporaries and by other people around him. So personally, it was very reassuring for me to see a mathematician with such innocence, such purity of mind, and such a broad outlook on mathematics, a eritable vision. So that encouraged me a lot to cultivate myself this global interdisciplinary approach to mathematics. So for me, that was really fundamental.

Nicolas Martin : And we will discuss, as Alain Connes pointed out earlier, topos which are a mathematical concept developed by Grothendieck which have been extremely criticized, and even more so , by the mathematical community, and of which all three of you were the continuators in a certain way. We will come back to this later, and I would like us to talk about this book, the circumstances in which it was written and then what it means to each of you. individually, because it's such a sum that there are many, many front doors and I would like to hear yours right away, after that.

(And we are therefore talking about *Récoltes et semailles,* the book-testament of Alexandre Grothendieck, a book which has just been published by Gallimard in two volumes, we are talking about it with Olivia Caramello, Alain Connes, Laurent Lafforgue .)

Before hearing about your relationship, your individual gateway to this book, I will read a very small excerpt. In the many introductions, propaedeutics, and introductory remarks that there are before getting to the heart of the book, if indeed we can speak of lively and ¸ca, I will let you say it, we are therefore page 99 of this new edition in the third chapter which is entitled *A letter.* This is what Alexandre Grothendieck writes:

> *“In this pre-letter, I would now like to* tell you (since it must be specified that it is therefore written in the first person and that there is an address in the second person in the singular to the reader.), *I would now like to tell you in a few pages (if possible) what is in question in* Récoltes et semailles, *to tell you in a more circumstantial way than does the only subtitle :* “Reflections and testimony on a mathematician's past”, *mine, from the past, you guessed it....*

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> *There are a lot of things in* Récoltes et semailles, and everyone *will probably see a lot of different things in it: a* ***journey,*** *the discovery of a past, a meditation on existence, a picture of mores of a milieu and an era, (or the picture of the shift, insidious and implacable from one era to another...); an investigation (almost police at times, and at others verging on the swashbuckling novel in the depths of the mathematical megalopolis...); a vast mathematical divagation (which will sow more than one...); a practical treatise on applied psychoanalysis (or, alternatively, a "psychoanalysis-fiction" book); a panegyric of self-knowledge; "My Confessions"; a diary ; a psychology of discovery and creation; an indictment (ruthless, as it should be...), even a settling of accounts in "the beautiful mathematical world" (and without giving gifts...).*

There are so many tracks, openings, which tell the richness of this work, I'm going to give you the floor, Alain Connes, which is in this list almost... here is, uh, programmatic, the one which is the yours, the gateway, your approach, which allows you to dive into this book?

Alain Connes : So from my own point of view, if you like, *Récoltes et semailles* played an extremely important role through the teaching I received from Grothendieck of the attitude of the researcher in his research. That is to say, if you like, in fact, Grothendieck, very far in *Récoltes et semailles,* manages to distinguish, if you like, roughly, two ways of doing mathematics : there's sports math, which is problem solving, and there's math which is... he distinguishes between yin and yang, but if you want some mathematics which is of a completely different nature, which consists in exploring a subject, a little, Grothendieck compares this exploration to the exploration of a woman's body, and he says, if you like, the whole attention it takes, all the care it takes, all the infinite patience it takes to do that. And, if you will, so what helped me enormously is that each of us is different, each mathematician is different, but in my own evolution, of course, if you want, at the start, when you want to be accepted as a mathematician, you don't have to, but it is better to solve problems that were posed, before, long before. So ¸ that is a kind of... how can I put it? of obligatory passage. But once this obligatory passage has been crossed, if we really want to do deep things, adopt this attitude which consists in trying to, if you want, to advance by ÿ in a given domain in applying techniques that are already known, that is not what is creative. And what Grothendieck explains wonderfully in his book is precisely, he gives, if you like, not a recipe, but a whole lot of reasons, a whole lot of explanations. For example, he says that you shouldn't be afraid of making mistakes, and that mistakes, on the contrary, are something positive, because when you understand that you've made a mistake, you understand that you is on the wrong track, well, that's where we progress. And so on.

And he explains, if you want, I want to say it, ¸ that, he explains that if, I will quote you his sentence because it is so true what he says, he says that he did not consider himself never as being more gifted than his contemporaries, absolutely not, what Grothendieck says is the following thing; he says :

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> *“I now see very clearly and beyond the slightest doubt this: that if, with intellectual gifts in no way exceptional* (he did not consider himself exceptional at all), *I could nevertheless constantly give my full measure in my mathematical work, and to produce a work and give birth to a vast, powerful and fruitful vision, it is to nothing but this fidelity that I owe, to this absence of any concern for conforming to norms, thanks to which I abandon myself with total confidence to the drive for original knowledge,* (¸that is extremely important, to say , and he even goes so far, if you like, as to link this drive, in the text, to the mother) *without cutting it or amputating it in any way that gives it its strength and its finesse and its nature. undivided.*

Olivia Caramello : Yes, on this subject, indeed, I would like to quote a passage which I find really magnificent, and which precisely illustrates the point which you have just raised. So, still in *R´ecoltes et semailles,* Grothendieck says:

> *“What makes the quality of the researcher's inventiveness and imagination is the quality of his attention, listening to the voice of things.”*

So precisely, he stresses the importance of having an attitude of receptivity, vis-à-vis the richness of the world. So the world is incredibly rich, and to be able to grasp this richness, you have to adopt a, let's say, very receptive attitude, and work on the quality of your own attention. So you can still see that this is an approach that is not at all the classic approach of mathematicians who are perhaps driven by the desire, say, to crack nuts, as one would say. Grothendieck *(laughs),* because he also compares a problem as if it were a nut that needs to be cracked, so most mathematicians would say “ok, let's try to crack this nut, we will use all the possible methods, it is not very important how we get there, for example, if we have a hammer available, we can use it, it is not a problem! ”. So he says “no, you have to be delicate, you have to listen to the voice of things”. And what does ¸that mean for him in fact, to use the image of the nut, ¸it consists in dipping this nut in an emollient liquid, and simply letting it rest, so that that, at the right moment, it opens naturally. So in fact, his mathematical approach has always been a global approach, aiming to put the problems in the most natural frame so that the difficulty dissolves naturally. So you see, it's a very original approach, it's magnificent.

Nicolas Martin : Laurent Lafforgue, your entry point, so I too will say a word for our listeners. Grothendieck says it, repeats it several times in *Récoltes et semailles,* he does not want this text to be addressed exclusively to mathematicians, he really makes it an important point, he writes “ what came up was kind of a long walk”, so he is very much on this idea of storytelling as you write, as you walk, “a long walk nade comment´ ee through my work as a mathematician, a walk intended above all for the layman, for those who have never understood anything about mathematics.”.

Laurent Lafforgue : Yes, so I had, let's say, two entry points into Grothendieck, and unlike Alain Connes, my first entry point into Grothendieck was really in his mathematical work. It was in a second time that I discovered *Récoltes et se meshes,* a few years later, and I must say that these two encounters, that is to say on board of

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the work and then of *R´ecoltes et semailles* were deeply moving for me.

The first because, therefore, I had started studying mathematics, I was at the Ecole Normale Supérieure, therefore, in mathematics, in fact because mathematics ematics were easy for me, even though my tastes were rather literary. I was more passionate about literature, and until then, I had not encountered in mathematics, or in science, something that seemed to me to be of a depth comparable to that of books. greatest literary works.
However, in the second year, at the Ecole Normale Sup´erieure, I learned of the existence of Grothendieck, on the occasion of a working group of initiation to geom´ algebraic etry. So I rushed to the Library, borrowed a number of volumes, started studying them, and was totally amazed. fact: it was mathematics like I had never seen, and, for the first time, in my studies, I had the feeling of being confronted with a work that was of a depth and beauty comparable to that of the greatest works of literature. So I can say that it was by reading the mathematical works of Grothendieck, which took me several years, that I understood that mathematics could be interesting, and even fascinating, a, that I understood the depth of mathematics.

And a few years later, when I was preparing a thesis, I heard about *Récoltes et semailles,* a copy was available at the University Library of Orsay, where where I was going. And I started to read it, in this library, and there too, I was totally amazed, because it was the same thing, it was It was about this mathematical work that I had already studied for several years, which for me was absolutely fascinating, and now the very author of this work was talking about it in personal terms, that is to say that it was no longer a mathematical work necessarily marble, impersonal, of course, always written without reference to any author, c It was the investment of a person, who had made this work possible, and an investment of incredible intensity. That is to say... Olivia quoted this passage from Grothendieck where he talks about the quality of attention, but indeed, this is what we perceive in him, that is to say the quality and intensity of the attention, to what extent he put himself at the service of things, and of the voice of things, and therefore it is what he was talking about in *R ´ecoltes et semailles.* And moreover, with another dimension that appears, which is his own literary dimension. In fact, Grothendieck is a very great writer, and I think that shows up in the quotes that have already been made. It was even heard just now, in the brief extract we heard, that is to say that Grothendieck expresses himself admirably, therefore even orally. And in writing, in fact, many of the passages in *Récoltes et semailles* are truly admirable on the literary level. They come under philosophy, they have a relationship with mathematics, but they also come under poetry. And besides, you should know that when the young Grothendieck was in high school, long before imagining that one day he would become a mathematician, his classmates and his teachers had given him a nickname. And the nickname they had given him was not at all “the Mathematician” or “the Mathematician”, it was “the Poet”.

Nicolas Martin : We are going to continue to dive into this work *Récoltes et semailles* and we are going to dive into it concretely, since in a few minutes, we are lucky to have had access not only to the typescript but also to the handwritten notes, of which there are literally tens of thousands, which are with a Parisian bookseller. We're going right after that, stay with us.

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(Musical interlude:) "Sentinelle mathematique" by Bertrand Burgalat, on France Culture, we are therefore talking about the work *Récoltes et semailles* by Alexander Grothendieck, which has just been published, this month of January, in two volumes, published by Gallimard, in the Tel collection, we talk about it with Olivia Caramello who is a mathematician and logician, Associate Professor at the Universit`a degli Studi dell'Insubria in Cˆome and holder of the Israel Gelfand Chair at the Institut des Hautes Etudes Scientifiques, at IHES, Alain Connes, mathematician, Emeritus Professor at IHES, holder of the Analysis and eometrics at the Coll`ege de France and winner of the Fields Medal in 1982, and Laurent Lafforgue, mathematician, winner of the Fields Medal in 2002, who now works at Huawei but on some of the mathematics developed by Alexander Grothendieck, we will talk about it in a moment).

Récoltes et semailles is a book, as we have said, which has long remained in boxes, shrouded in a certain aura of mystery. Besides the over 1000 pages of original typescript, there are also tens of thousands of pages of notes. Hello Celine Loozen.

C´eline Loozen : Hello Nicolas and hello to all.

Nicolas Martin : You had the chance and the privilege of going to consult these handwritten notes which are still sleeping in the cellar of a Parisian bookseller.

C´eline Loozen : And yes, thanks to Jean-Bernard Gillot who is like the guardian of the Grothendieck archives. In fact, after his death, his son Matthieu asked him to appraise the entire corpus, thousands of pages, left in crates and boxes. So, together, they will bring everything back to Paris one night in November 2014, by car. After having estimated their value, although priceless, these texts still remain in the cellar of his Alain Brieux bookstore.
Together we went through a few excerpts from *Récoltes et semailles* but also notebooks, finely scribbled pages, sometimes recounting complex mathematics, personal reflections, or family memoirs.

(Noise of the entrance door in the Alain Brieux bookstore).

Jean-Bernard Gillot : So, we are in the Alain Brieux bookstore, rue Jacob in Paris. My name is Jean-Bernard Gillot and therefore, a few years ago, I was asked to take care of the manuscripts of Alexander Grothendieck. So there are 60,000 pages which are in trunks, which we will go and see afterwards. And then there, for the moment, there was a box, which was a box from the Encyclop´edie Universalis, which is still marked Alexander Grothendieck and we searched everything, we found everything. Presented, we put everything, on paper, in little boxes, everything laid flat. So we have manuscripts that I'm going to show you. Now I will show you the little boxes.

C´eline Loozen : And so it is mainly thanks to you that we recovered these manuscripts which allowed the publication of the sequel, years later.

Jean-Bernard Gillot : Yeah, I'm the goalkeeper, that's it. I am the guardian.

C´eline Loozen : The guardian of *Harvests and sowing...*

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Jean-Bernard Gillot : ...of *R´ec...* of all the manuscripts.

C´eline Loozen : So there, yes, everything is kept in crates.

Jean-Bernard Gillot : So there, we put everything, we have 1500 pages, which are only drafts in all directions, it was “in the trash”, eh, in quotation marks, it was something something that was in the trash.

C´eline Loozen : Ah yes, it is said that Alexander Grothendieck wanted to throw everything away, put everything in the trash, his entire production...

Jean-Bernard Gillot : That's it, except that in fact he didn't want to throw anything away, the proof is that it's all there. So here it is...

C´eline Loozen : So among these funds, there are in particular writings found in *R´ecoltes et semailles.*

Jean-Bernard Gillot : Possible, probably, but there we also have things, here he was writing, he was someone who was crazy about writing, so he was writing all the time. So there, there is a letter...

C´eline Loozen : What we can already say is that it is very heterogeneous, in what we see, there are equations, there are formulas, and then there are also spurts of sentences, little diagrams too.

Jean-Bernard Gillot : And so all the documents are kept, we kept everything, there must be one or two tax letters and he writes something behind. Here, there, for example, these are mathematics. I know there is physics, there are lots of things. So there you have it, 1500 pages.

C´eline Loozen : There is also poetry in *R´ecoltes et semailles.*

Jean-Bernard Gillot : There is also poetry, but it is enormous. There then we are going to see after the rest of the pages, so we have approximately 60,000 pages which are there. Anyway, I don't know, I mean, we were talking about topos earlier, there, there are extremely important things about topos, it's the future, it's Huawei who will work on them. , it's extraordinary.

C´eline Loozen : It is true that one of the central mathematical objects described in *Harvesting and meshing* is indeed this object, the topos, which are a very powerful and unifying tool in the field of mathematics.

Jean-Bernard Gillot : *(laughing)* That's what I thought I understood. Here it is. Alright now, let's...

C´eline Loozen : In any case, he said it was one of his finest discoveries.

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Jean-Bernard Gillot : I'll show you the rest.

C´eline Loozen : We go down to the basement.

Jean-Bernard Gillot : We descend into the den of dens, that's it. We have 5 canteens, which are all referenced. So the canteens, they are in boxes, there are 48 boxes, or a little more probably. Each box is a binding that Alexander had made by a friend of his who was a bookbinder, who had learned bookbinding in prison, he is someone from the Direct Action group who has no blood on his hands. , but who had been close to Direct Action. So we have... Everything is referenced, everything is dated, everything is numbered.

C´eline Loozen : *(exclaiming)* This fine handwriting...

Jean-Bernard Gillot : Exactly, and so here we have mathematics, physics, lots of things.

C´eline Loozen : Oh my! It seems that he is writing in the flow of his thoughts, as if he could never stop, that's really the feeling it gives.

Jean-Bernard Gillot : I think that was the goal of his life, it was to write, as he says “we don't buy books, we make them”. So he charges, he overloads, he erases, he rectifies, but everything is intact, that's it. And so there's about 60,000 pages here, one of the last packages. Because in the end, he gets angry with the...

C´eline Loozen : ... the community...

Jean-Bernard Gillot : No, the community, he spent his life getting angry with the community, if I understood correctly. He gets angry with his friend who was the bookbinder, he says to him “I will give you the boxes”, he refused, so they got angry, so in the end, he no longer has no one to take care of his boxes. So he continues his work and then he puts it in boxes.

So there, we are probably in things that are close to God, to the Devil, finally...

C´eline Loozen : It is true that there is a lot of metaphysics and spirituality.

Jean-Bernard Gillot : Absolutely, and then there are also family stories. It questions a lot of things. Voil`a, voil`a, ¸ca remains an incredible object, and I am guardian of ¸ca. I don't even understand why this is my home *(laughing).*

C´eline Loozen : Do you think that the publication of this opus, some forty years after it was written, will change anything?

Jean-Bernard Gillot : Yes, it's the beginning of Alexander Grothendieck's story, I think, between that, the topos, and... it's the beginning of a bigger adventure . We are dealing with a genius, and then there he is, his manuscripts are there at the back of my cellar, it's ridiculous. But hey, there they are. I protect them

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for the moment. I can't wait for it to go away and for everyone to benefit.

Nicolas Martin : Here are those tens of thousands of manuscript pages by Alexander Grothendieck in the cellar of this Parisian bookseller.

A reaction, indeed, should something be done? Do we have to take them, protect them, put them elsewhere, remove them, since apparently, this gentleman is only asking for that, to give them to the common knowledge? Laurent Lafforgue?

Laurent Lafforgue : The answer is “yes, of course!”. These are very valuable texts from one of the greatest creators of the 20th century, one of the greatest scientific minds in history, so it is obviously valuable, it These manuscripts would indeed have to be saved, they would also have to be scanned, to be made available to all. Having said that, I would also like to add a remark, inspired by the simple fact that here, there are 60,000 pages, one can ask oneself “how is this possible?”. But here, it is necessary to make a comment, which is that for Grothendieck, writing is the means of the search for truth. So Grothendieck, in fact, did not think with his head as they say, he wrote down everything, he wrote down everything that came to his mind and he spent his life doing that, so mainly when it comes to mathematics, but not only. And in fact, in *Récoltes et semailles,* he insists a lot on what he calls the creative power of writing. And this is something that he experienced, and which in fact is a lesson for everyone.
That is to say that by writing, one finds much more, that is.

Nicolas Martin : We heard about topos in this report, it's very important, I'd like us to talk about it now, it's the great creation, what Grothendieck said he was most proud of. to have created a unifying desire for mathematics, and yet, paradoxically, for reasons that you may be able to explain to me, one or the other, these mathematics a have very bad press. Today in the mathematical community, it is difficult to carry out research and work on topos. Alain Connes, you gave lessons to the Coll`ege de France on topos but it did not last very long. You yourself, Olivia Caramello, it was difficult, and you were pressured not to continue on this path, how is that all explained? *(Alain Connes signals me no).*

Alain Connes : Yes. No, no, in fact, if you want, no. That is a completely external version of reality: the reality, if you will, is that the conceptual notion of topos...

Nicolas Martin : ... that you can briefly remind us of, please.

Alain Connes : Ah, I can remind you what it is. In fact, basically, if you like, we had, before Grothendieck, the habit, to study a space, everyone knows that the role of space, if you like, is essential in geom´ etrie, in mathematics. Before Grothendieck, when we wanted to know a space, we looked at it directly, okay? And we were trying to figure it out. What the idea of the topos does, which is a wonderful idea, is... if you like, it puts the space behind the scenes, and what we do is math. ordinary ematics with a parameter, this parameter is in the space in question, it is behind the scenes. To give you the simplest possible example, suppose that the space in question is simply two points.

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Well, the math you're doing is you're doing twice the math that's ordinary set theory. Well, what is wonderful, if you like, in the theory of topos, which is there, is that it has two characteristics, the first is that, in precisely , by analyzing what happens in the ordinary context of set theory, but done with parameters in the topos, we arrive at a knowledge of this space which is the space of parameters, which is the topos, much finer than if we had looked at it directly. And I have to say, if you like, that I can't help saying that right now I'm writing a book with a psychoanalyst, which is Patrick Gauthier-Lafaye, in which we use this metaphor, but in relation, precisely, to psychoanalysis.

And the second thing, which is absolutely extraordinary, if you like, in this idea of the topos, is that it amounts to what? It amounts to looking at the mathematician at work in a structuralist way. That is, the mathematician at work will manipulate sets. But the structura list doesn't care that they are sets. He will look at the mathematician who manipulates objects and arrows. And he will say “this mathematician is working on what in mathematics is called a category”. And this structuralist is going to say “but what are the properties of this category which enable the mathematician to work?”. Well there, we are at the heart of the topos.

Nicolas Martin : How to explain, then, Olivia Caramello, that the topos, so obviously correct me if I describe it in a way that is too caricatural, but have such bad press, or are a field of work for mathematics ematics which were ultimately rejected or rejected by the institutions?

Olivia Caramello : Yes, well, in fact, I did a lot of thinking on my own, reading *Récoltes et semailles* and also, really, on the basis of the one that was my own experience. life experience, because indeed, I received a lot of opposition, in fact, since the beginning of my career, quite simply because I wanted to develop, to a global and systematic way this theory, precisely with the aim of realizing this aspiration of unification which had already been expressed by Grothendieck, notably in *Récoltes et semailles .* So in fact, all my research work has been directed towards the goal of developing techniques, methods, for transferring knowledge between completely different parts of mathematics. ematics, through topos. So in fact, topos can be used incredibly effectively, like bridge objects, to link the most diverse mathematical contexts to each other. So in fact, we can metaphorically think of a topos as a place in which different points of view meet, reflecting each other. So, I give this metaphor to underline, to really bring forward this aspect of unification because I think it's the one that has really engendered the most hostility. I think it is not the technicality of topos as a mathematical object like other objects, because there is of course a whole technicality, even, the theory is all the same very sophisticated and very deep on a purely technical level. But it is not the technical aspect which was at the origin of the ostracism. I think it's really this global and inter-disciplinary dimension that bothers people, because today, let's say, mathematics has become hyper-specialized so each specialist works in his corner, with his own methods, he gets used to thinking in a certain way. However, with these bridges that we manage to generate with the topos, we can manage in particular to demonstrate a result in a sector of mathematics by using completely foreign methods. to that sector. One can

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succeed in establishing bridges between sectors that are completely distant in appearance, and therefore, we can arrive at a specialist in a certain field with a result that surprises him very much, that we manages to demonstrate by methods which are not his own. So you can already understand that ¸ca, ¸ca can be worrying for some people, if you don't have enough open-mindedness to accept this plurality of points of view. So I think there's a certain dogmatism in certain mathematical circles that makes you get used to a certain language, and then you sort of close yourself off, after years. of hyper-specialization, because you still have to understand that working in any area of mathematics today requires a colossal technical investment. So, it is all the same humanly understandable that we are very fond of certain methods and afterwards, we say “Well, no! I don't want to see anything else.' is not mine...

Nicolas Martin : For example?...

Olivia Caramello : The theory of models, for example, I happened to demonstrate a result among my first, where I made a broad generalization of the Fra¨ÿss´e eorem, in model theory, which is a very important result. And in fact, in the audience, I remember well, an important model theorist who could not believe that my result was correct, because it was too general. And in fact, he spent the whole afternoon trying to find a counterexample, of course, without succeeding, because my proof was absolutely correct, except that 'It was formulated in a language that he... He told me, “No, but I'm not even starting to try to understand, because I'm not going to succeed anyway. ” He told me like that. And so, he preferred to spend 4 hours of his time, and he also tormented me because I was there, and therefore, he tried to fabricate all his counter-examples for me and ... *(laughing),* ¸it was quite painful but, it's just to give you an idea. So I think that there is really this interdisciplinary aspect that is disturbing. Then there are many other aspects, of course too.

Nicolas Martin : Laurent Lafforgue, one word because you left the academic environment to go into the private sector and because in the private sector, in this case, we works... in this case your employer works, asks you to work on the topos and uses this tool, which seems to be particularly effective, for applied work, which seems to to be exciting and effective work.

Laurent Lafforgue : Yes, it's a story that is totally amazing for me, that I would never have imagined a few years ago. That is to say that for about ten years, in fact since I have known Olivia Caramello and her work, I have become in the academic world a fervent supporter of the development of thesis. theory of topos. And, like all the people who wanted to develop topos or contribute to their development, I came up against very great hostility and, to my total surprise, I I therefore found in an environment of engineers, therefore in this case the firm Huawei, in France, therefore I found, therefore among these engineers ears much more favorable. So it's something I would never have expected, which amazes me to this day. And so today, finally for a few months, I left the academic world, I am at Huawei, and therefore my environment is made up of engineers and managers of the hierarchy of the research of Huawei who are totally favorable to the development of topos,

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who think today, that is to say only a few years after having learned of the existence of this theory, that it is extremely important, and, therefore, some of them think that the topos are going to become... the Grothendieck topos are going to become, or can become the mathematics of artificial intelligence, so that is to say something of absolutely colossal importance, and that's it. So, for me, it's unimaginable because it's been 60 years since the theory of topos was introduced by Grothendieck, it was developed by him already at the length of hundreds and hundreds of pages, which he himself, who everyone knows is one of the greatest scientific geniuses in history, has insisted enormously on the power about topos, about the importance of topos, even about the importance of topos beyond mathematics. So, in fact, in *Récoltes et semailles,* a certain number of pages are devoted to this, Grothendieck says why topos are so important to him, he says it in terms that mathematicians can understand but also, that even a reader who does not know mathematics can be sensitive to the beauty and depth of what Grothendieck says when he talks about topos. So he wrote those pages, and it had no effect in the academic world. So there is a mystery there, which Grothendieck himself cannot explain, he observes this hostility, he does not understand it. Olivia has just proposed elements of explanation, but, for me, it remains a mystery. That is, in fact, topos are a sensitive subject. And it's weird because usually, when we say a sensitive subject, we understand that a sensitive subject is, for example, a political subject on which people do not agree. We do not understand that a scientific subject, a theoretical definition can be a sensitive subject. But in fact, it is. So there you have it, it's a fact, which I personally cannot explain, or at least not in a satisfactory way for me.

Nicolas Martin : Well listen, what I propose to you since, as you have heard, Grothen dieck is always fascinating, *Récoltes et semailles* is absolutely captivating, and in one hour, we barely had the time to touch on a certain number of subjects, so I suggest, if you don't mind, that we redo a second one, since in the end there are two volumes *of Récoltes et semailles* at Gallimard, which we are devoting a second program, we will talk about the division into two volumes if you want, that is one of the subjects that annoys us, but we will dedicate a second program to continue this quite fascinating discussion . Thank you very much to all three. Thank you Alain Connes, thank you Olivia Caramello, thank you Laurent Lafforgue. *Récoltes et semailles,* it is therefore in two volumes at Gallimard in the collection Tel [1].

1The scientific method is C´eline Loozen, Natacha Triou, No´emie Naguet de Saint-Vulfran, Antoine Beauchamp, Mariam Ibrahim, Amel Boucherka, Olivier B´etard at the production, Ludovic Auger at technique, and as always in podcast on the Radio-France application and on Franceculture.fr forever until the end of time, until proven otherwise.