

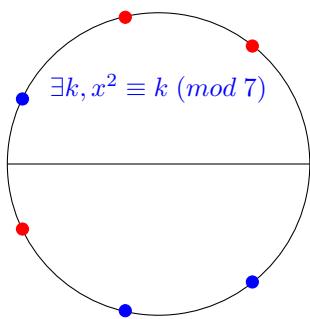
Résidus quadratiques sur colliers (Denise Vella-Chemla, 22.5.2019)

Un nombre premier  $p$  est caractérisé par le fait que dans  $\mathbb{Z}/p\mathbb{Z}$ ,  $\frac{p-1}{2}$  nombres sont résidus quadratiques et  $\frac{p-1}{2}$  ne le sont pas. Dans une représentation des classes de congruences sur un cercle, les résidus quadratiques sont symétriques ( $x$  résidu de  $p \iff n-x$  résidu de  $p$ ) pour les nombres premiers de la forme  $4k+1$  et anti-symétriques ( $x$  résidu de  $p \iff n-x$  non résidu de  $p$ ) pour les nombres premiers de la forme  $4k+3$ .

Solutions de  $\exists k, x^2 \equiv k \pmod{7}$  : 1, 2, 4.

7 est premier.

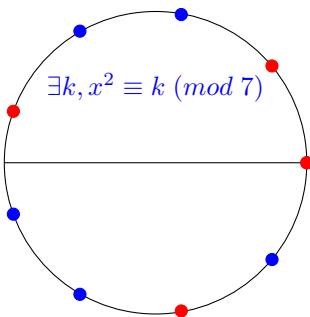
Il y a 3 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{9}$  : 0, 1, 4, 7.

9 est composé.

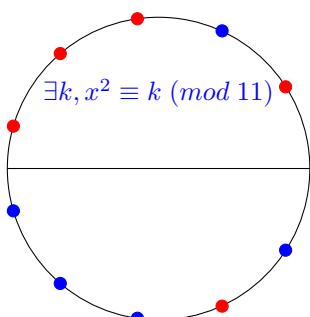
Il y a 4 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{11}$  : 1, 3, 4, 5, 9.

11 est premier.

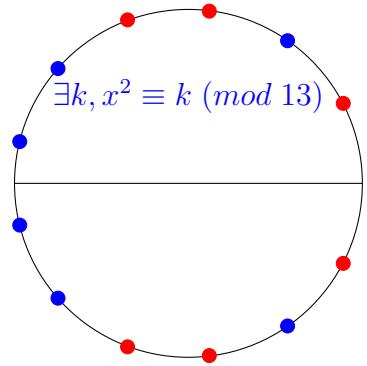
Il y a 5 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{13}$  : 1, 3, 4, 9, 10, 12.

13 est premier.

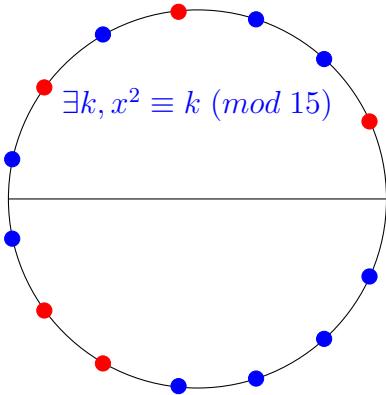
Il y a 6 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{15}$  : 1, 4, 6, 9, 10.

15 est composé.

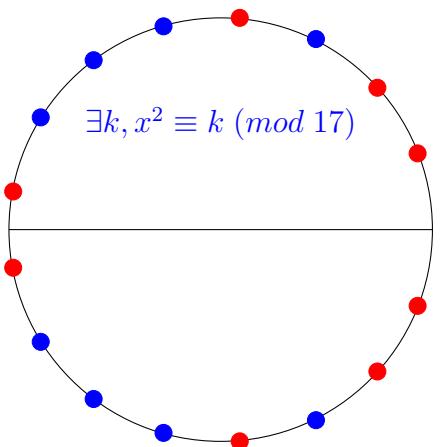
Il y a 5 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{17}$  : 1, 2, 4, 8, 9, 13, 15, 16.

17 est premier.

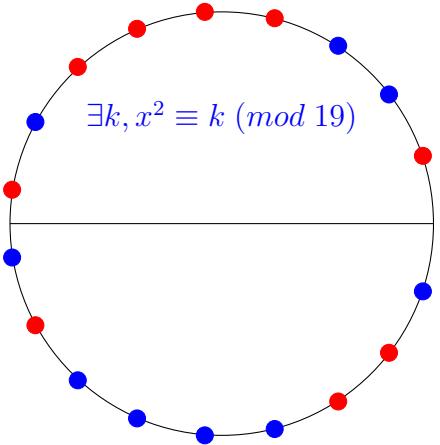
Il y a 8 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{19}$  : 1, 4, 5, 6, 7, 9, 11, 16, 17.

19 est premier.

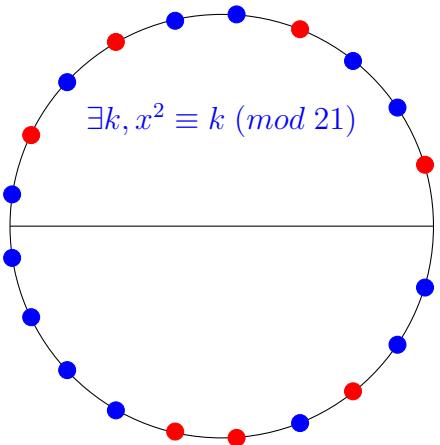
Il y a 9 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{21}$  : 1, 4, 7, 9, 15, 16, 18.

21 est composé.

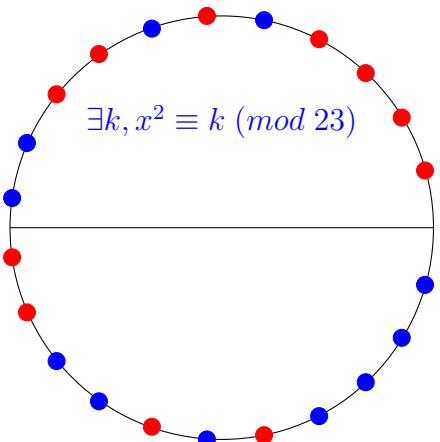
Il y a 7 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{23}$  : 1, 2, 3, 4, 6, 8, 9, 12, 13, 16, 18.

23 est premier.

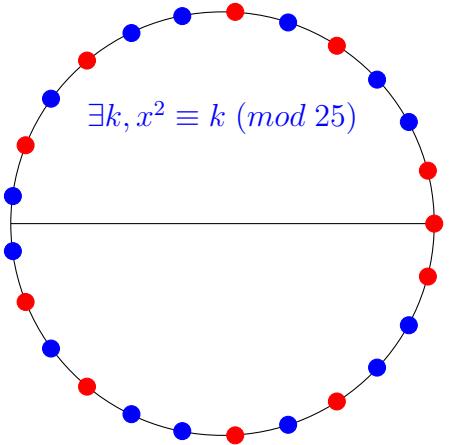
Il y a 11 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{25}$  : 0, 1, 4, 6, 9, 11, 14, 16, 19, 21, 24.

25 est composé.

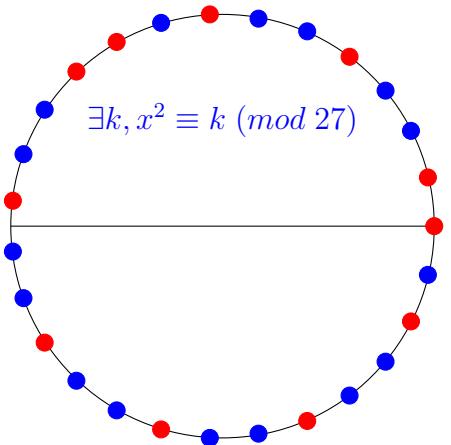
Il y a 11 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{27}$  : 0, 1, 4, 7, 9, 10, 13, 16, 19, 22, 25.

27 est composé.

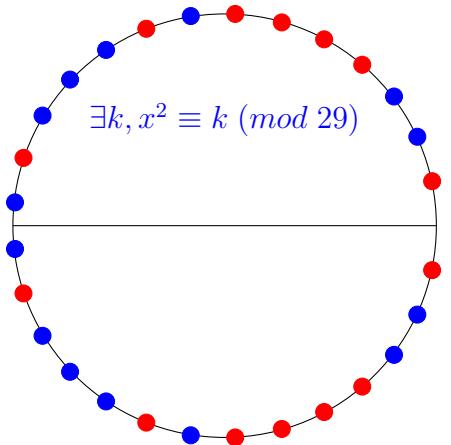
Il y a 10 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{29}$  : 1, 4, 5, 6, 7, 9, 13, 16, 20, 22, 23, 24, 25, 28.

29 est premier.

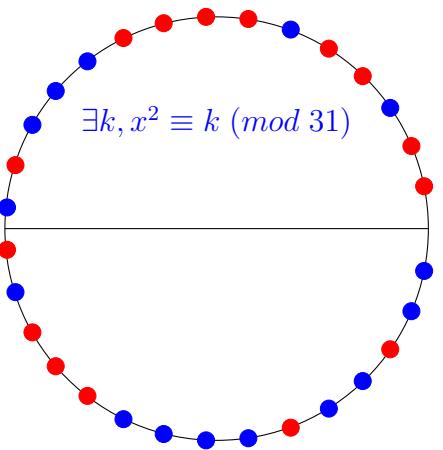
Il y a 14 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{31}$  : 1, 2, 4, 5, 7, 8, 9, 10, 14, 16, 18, 19, 20, 25, 28.

31 est premier.

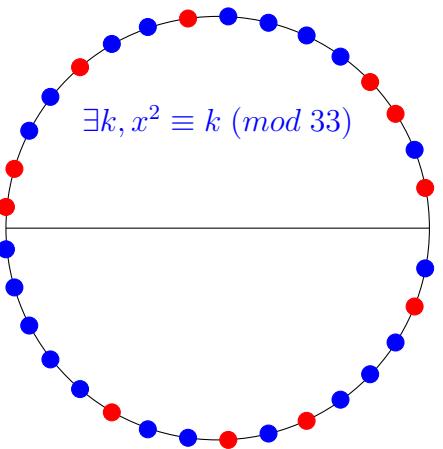
Il y a 15 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{33}$  : 1, 3, 4, 9, 12, 15, 16, 22, 25, 27, 31.

33 est composé.

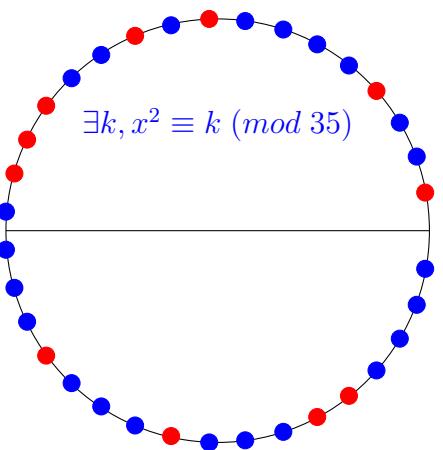
Il y a 11 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{35}$  : 1, 4, 9, 11, 14, 15, 16, 21, 25, 29, 30.

35 est composé.

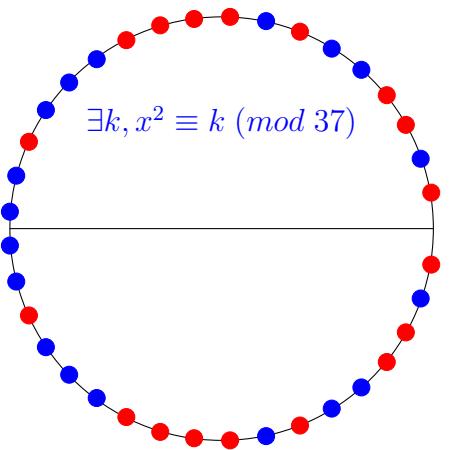
Il y a 11 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{37}$  : 1, 3, 4, 7, 9, 10, 11, 12, 16, 21, 25, 26, 27, 28, 30, 33, 34, 36.

37 est premier.

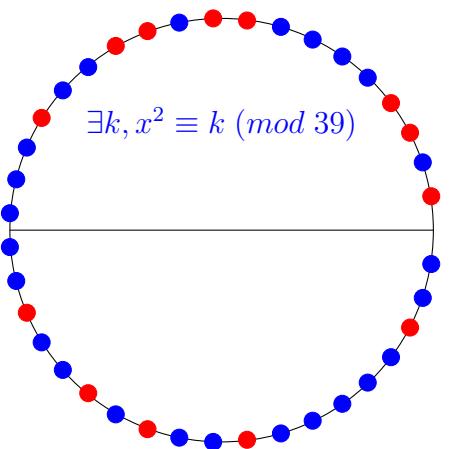
Il y a 18 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{39}$  : 1, 3, 4, 9, 10, 12, 13, 16, 22, 25, 27, 30, 36.

39 est composé.

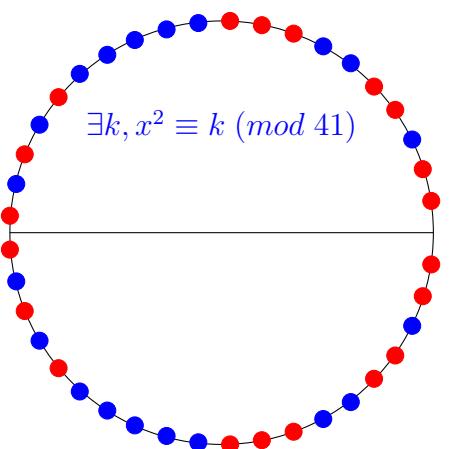
Il y a 13 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{41}$  : 1, 2, 4, 5, 8, 9, 10, 16, 18, 20, 21, 23, 25, 31, 32, 33, 36, 37, 39, 40.

41 est premier.

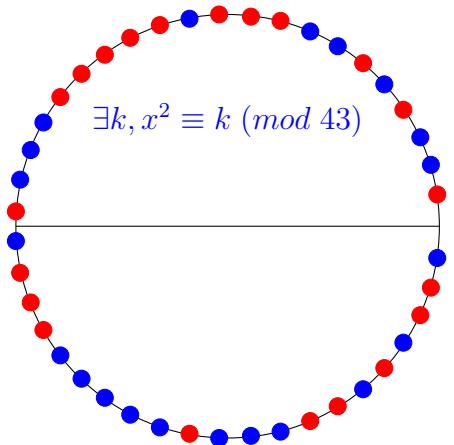
Il y a 20 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{43}$  : 1, 4, 6, 9, 10, 11, 13, 14, 15, 16, 17, 21, 23, 24, 25, 31, 35, 36, 38, 40, 41.

43 est premier.

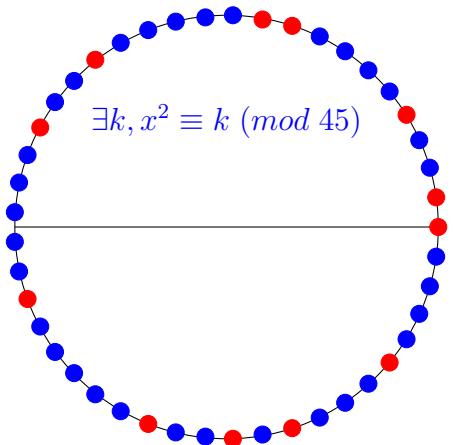
Il y a 21 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{45}$  : 0, 1, 4, 9, 10, 16, 19, 25, 31, 34, 36, 40.

45 est composé.

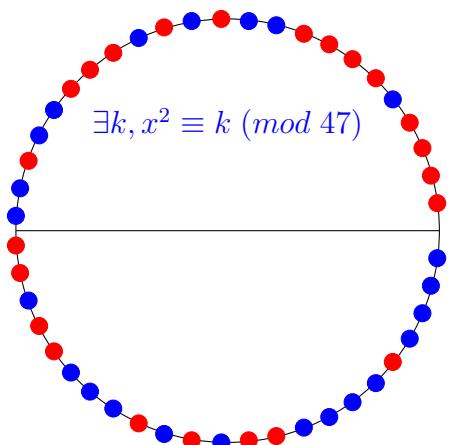
Il y a 12 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{47}$  : 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 17, 18, 21, 24, 25, 27, 28, 32, 34, 36, 37, 42.

47 est premier.

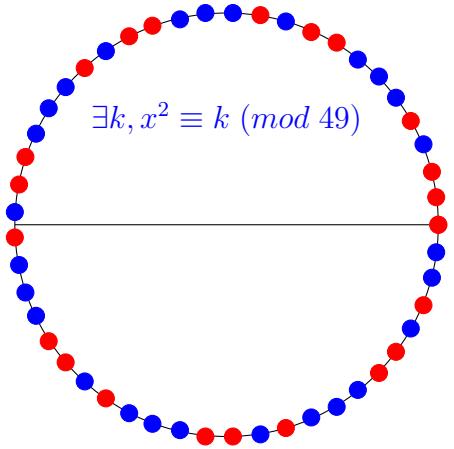
Il y a 23 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{49}$  : 0, 1, 2, 4, 8, 9, 11, 15, 16, 18, 22, 23, 25, 29, 30, 32, 36, 37, 39, 43, 44, 46.

49 est composé.

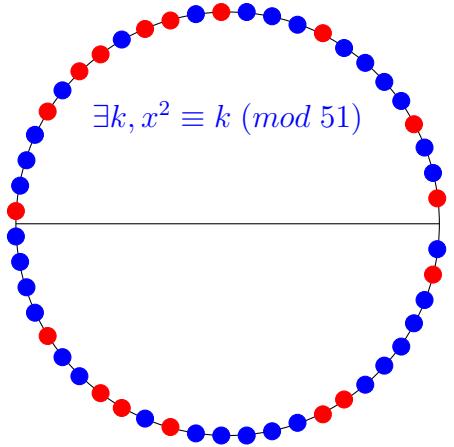
Il y a 22 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{51}$  : 1, 4, 9, 13, 15, 16, 18, 19, 21, 25, 30, 33, 34, 36, 42, 43, 49.

51 est composé.

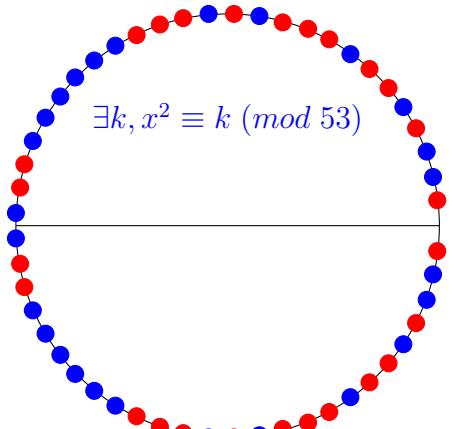
Il y a 4 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{53}$  : 1, 4, 6, 7, 9, 10, 11, 13, 15, 16, 17, 24, 25, 28, 29, 36, 37, 38, 40, 42, 43, 44, 46, 47, 49, 52.

53 est premier.

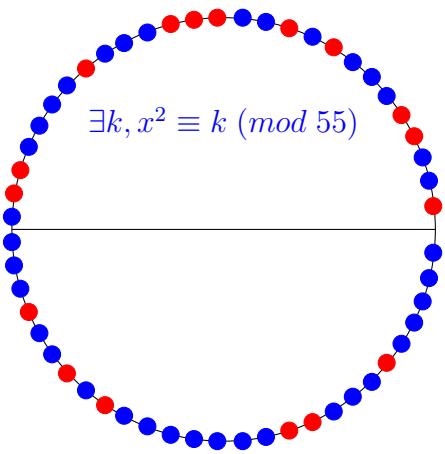
Il y a 4 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{55}$  : 1, 4, 5, 9, 11, 14, 15, 16, 20, 25, 26, 31, 34, 36, 44, 45, 49.

55 est composé.

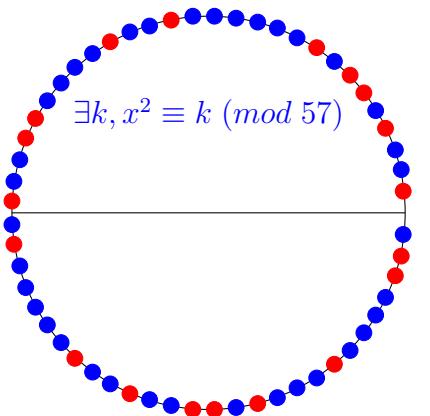
Il y a 4 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{57}$  : 1, 4, 6, 7, 9, 16, 19, 24, 25, 28, 30, 36, 39, 42, 43, 45, 49, 54, 55.

57 est composé.

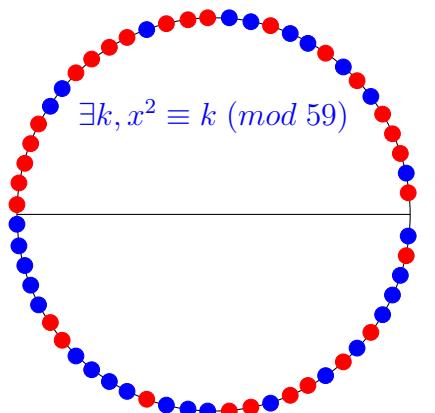
Il y a 4 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{59}$  : 1, 3, 4, 5, 7, 9, 12, 15, 16, 17, 19, 20, 21, 22, 25, 26, 27, 28, 29, 35, 36, 41, 45, 46, 48, 49, 51, 53, 57.

59 est premier.

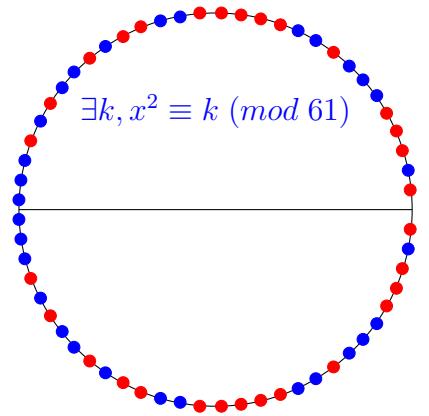
Il y a 29 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{61}$  : 1, 3, 4, 5, 9, 12, 13, 14, 15, 16, 19, 20, 22, 25, 27, 34, 36, 39, 41, 42, 45, 46, 47, 48, 49, 52, 56, 57, 58, 60.

61 est premier.

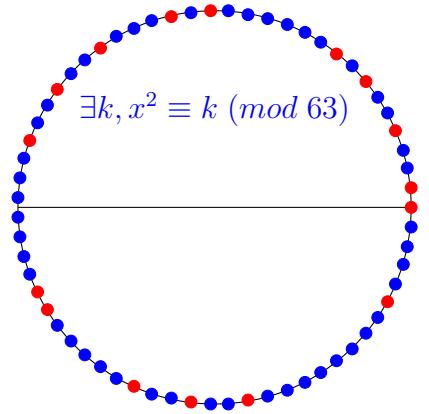
Il y a 30 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{63}$  : 0, 1, 4, 7, 9, 16, 18, 22, 25, 28, 36, 37, 43, 46, 49, 58.

63 est composé.

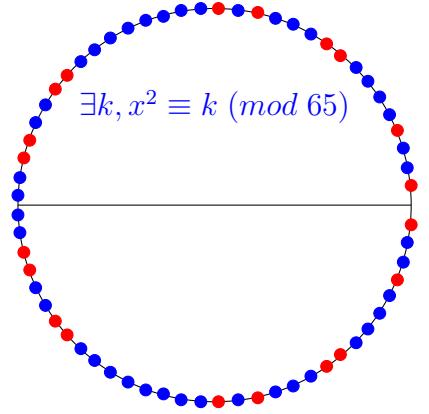
Il y a 16 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{65}$  : 1, 4, 9, 10, 14, 16, 25, 26, 29, 30, 35, 36, 39, 40, 49, 51, 55, 56, 61, 64.

65 est composé.

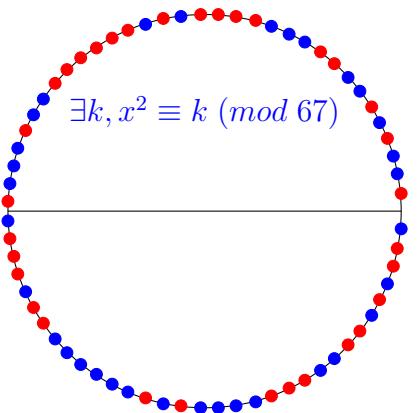
Il y a 20 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{67}$  : 1, 4, 6, 9, 10, 14, 15, 16, 17, 19, 21, 22, 23, 24, 25, 26, 29, 33, 35, 36, 37, 39, 40, 47, 49, 54, 55, 56, 59, 60, 62, 64, 65.

67 est premier.

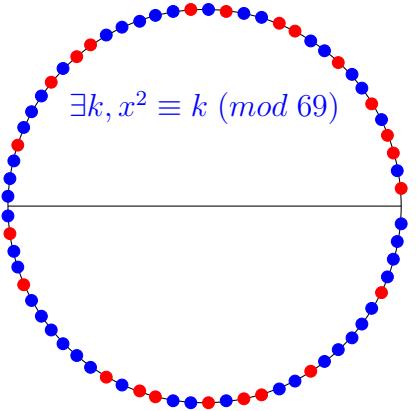
Il y a 33 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{69}$  : 1, 3, 4, 6, 9, 12, 13, 16, 18, 24, 25, 27, 31, 36, 39, 46, 48, 49, 52, 54, 55, 58, 64.

69 est composé.

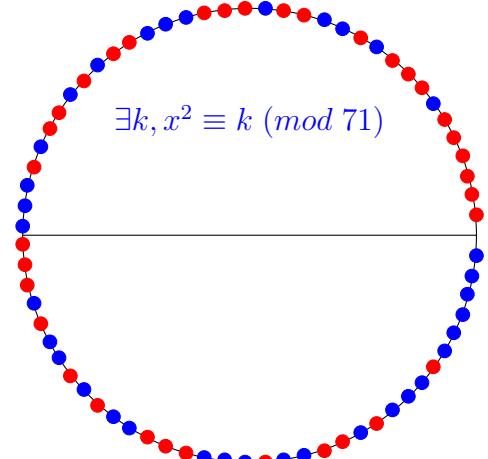
Il y a 23 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{71}$  : 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 19, 20, 24, 25, 27, 29, 30, 32, 36, 37, 38, 40, 43, 45, 48, 49, 50, 54, 57, 58, 60, 64.

71 est premier.

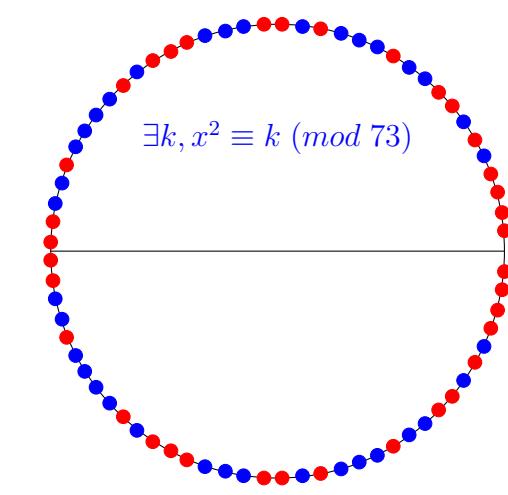
Il y a 35 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{73}$  : 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 19, 23, 24, 25, 27, 32, 35, 36, 37, 38, 41, 46, 48, 49, 50, 54, 55, 57, 61, 64, 65, 67, 69, 70, 71, 72.

73 est premier.

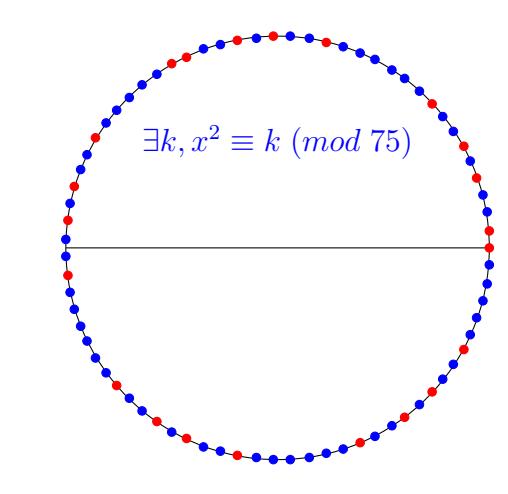
Il y a 36 solutions.


$$\exists k, x^2 \equiv k \pmod{73}$$

Solutions de  $\exists k, x^2 \equiv k \pmod{75}$  : 0, 1, 4, 6, 9, 16, 19, 21, 24, 25, 31, 34, 36, 39, 46, 49, 51, 54, 61, 64, 66, 69.

75 est composé.

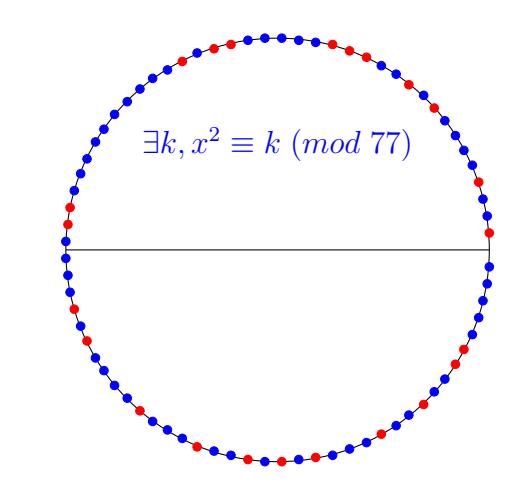
Il y a 22 solutions.


$$\exists k, x^2 \equiv k \pmod{75}$$

Solutions de  $\exists k, x^2 \equiv k \pmod{77}$  : 1, 4, 9, 11, 14, 15, 16, 22, 23, 25, 36, 37, 42, 44, 49, 53, 56, 58, 60, 64, 67, 70, 71.

77 est composé.

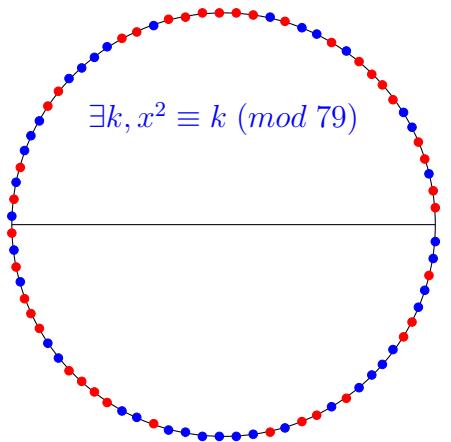
Il y a 23 solutions.


$$\exists k, x^2 \equiv k \pmod{77}$$

Solutions de  $\exists k, x^2 \equiv k \pmod{79}$  : 1, 2, 4, 5, 8, 9, 10, 11, 13, 16, 18, 19, 20, 21, 22, 23, 25, 26, 31, 32, 36, 38, 40, 42, 44, 45, 46, 49, 50, 51, 52, 55, 62, 64, 65, 67, 72, 73, 76.

79 est premier.

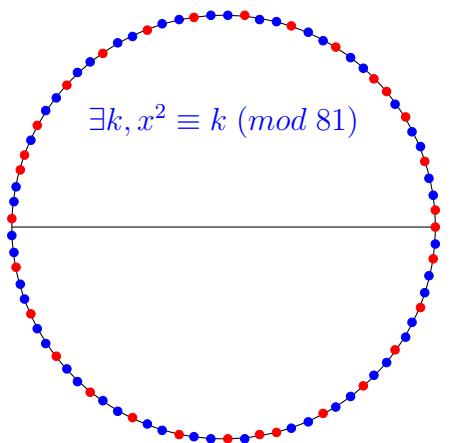
Il y a 39 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{81}$  : 0, 1, 4, 7, 9, 10, 13, 16, 19, 22, 25, 28, 31, 34, 36, 37, 40, 43, 49, 52, 55, 58, 61, 63, 64, 67, 70, 73, 76, 79.

81 est composé.

Il y a 30 solutions.



Solutions de  $\exists k, x^2 \equiv k \pmod{91}$  : 1, 4, 9, 14, 16, 22, 23, 25, 29, 30, 35, 36, 39, 42, 43, 49, 51, 53, 56, 64, 65, 74, 77, 78, 79, 81, 88.

91 est composé.

Il y a 27 solutions.

