An interview with Alain Connes

1 Noncommutative geometry

The subject that has occupied me for all these years is very, very far from being exhausted. It is a subject that begins with the discovery of Heisenberg. It is a discovery of physics in the years 25, 1925 of course, and what Heisenberg discovered is something quite extraordinary. He discovered that when we do physics with microscopic systems, well, we can no longer do calculations as we are used to, that is to say what is called commutative algebra.

We can no longer use commutativity. So commutativity means that if you write mc^2, or c^2 times m, it's the same thing, but when you do quantum physics, you can't. So what does that mean ? It means that it is essential too, as much for physics than for mathematics, to understand more subtle spaces which are noncommutative spaces. So if it was only... if noncommutative geometry was only a generalization of geometry to spaces in which the coordinates do not commute, that would not be very interesting.

What I had discovered in my thesis was that, precisely, a noncommutative algebra by the simple non commutativity generates its own time, i.e. evolves over time. It's something which is difficult to explain, but which has a depth, that is to say that basically, we can summarize it in the following form. We can say, if you want, that commutative algebra is static, it doesn't move and noncommutative algebra evolves. So you have to understand that when we talk about noncommutative algebra, non-commuting coordinates, etc., you might think, a little simplistically if you like, that it's a mathematical abstraction that has nothing to do with our habits, etc. In fact, this is not at all the case because, precisely, the non-commutativity, we are extremely familiar with This is because when we write, with letters, when we write words, sentences, etc., we must of course pay attention to the order of the letters. In written language, you cannot swap letters.

If you have fun swapping letters, you get what is called an anagram. And obviously, at that time, we can have two sets of letters which are the same in the commutative frame, but which have completely different meanings in the noncommutative case. The example that was the occasion of a book that we wrote recently with Jacques Dixmier and my wife, it's if you want, this magnificent anagram which is due to Jacques Perry-Salkow and which is, precisely, *"L'horloge des anges ici-bas"*[1], which has to do with time and the anagram of that is *"Le boson scalaire de Higgs"*[2].

So, we can see that if we only look at the commutative part, if you will, of these two sentences, they are the same, but on the other hand, they are not at all the same, they have not at all the same meaning. The quantum, the great discovery of Heisenberg, is that, precisely, it is necessary to be careful. The way this noncommutative geometry has evolved, and that's why it is very, very far from being exhausted is that, on the one hand, there is a very, very strong link with physics. So here I have it developed during numerous courses, with my collaborators, etc. So there is a link with the fact that it is precisely the formalism of quantum mechanics that allows us to understand how continuous variables can coexist with discrete variables, and how we can reformulate geometry, Riemannian geometry, in a form which is much more compatible with quantum than is general relativity.

So that is a whole area, it is a whole area which is still open, which is far from being exhausted. There has been great progress. And then there was another extremely exhilarating episode that happened, is that if you want, this noncommutative geometry, precisely, allows you to encode spaces which, normally, for mathematicians, appear as very, very singular spaces. Those are quotient spaces, but these are spaces that we meet in mathematics in fact, very, very often, people don't realize it because as soon as we take what we call in mathematics an inductive limit, we will fall on a space which is of this nature, because it is defined as a quotient space. And the idea, the basic idea, is that when we take a quotient that is hard to take, we have not to look at it as a whole. But you have to look at it like a noncommutative space where non-commutativity comes from the fact that we will identify between them points which are distinct and

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1. the Clock of the angels down here
2. The Higgs scalar boson

so we're going to have arrows, etc. And that's what makes it noncommutative.

So there was an episode completely... which is far from over of course, it's actually a space fundamental to number theory, which is actually connected to prime numbers, which is a noncommutative space. So there is a very, very long development that has taken place, which has corresponded to many of courses I have given, etc. but which continues to evolve.

And now, we found with Katia Consani, we found very, very recently that in fact, there was an object of very, very pure algebraic geometry which only involves integers with the three operations of *inf* of two numbers, of the sum of two numbers and their product, but which involves two fundamental concepts, the concept of topos, which is due to Grothendieck and the concept of algebra of characteristic 1. This is yet another story.

This object means that we have exactly the parallel with what André Weil had done in algebraic geometry, precisely to deal with a fundamental problem in finite characteristic.

What you are telling us there seems to demonstrate that mathematics generates other mathematics.

Yes, but it's not that they generate, no. What you have to understand is that I always had this long discussion with Jean-Pierre Changeux. And it's not that you beget, no, it's like if..., but let me explain why it is not that one begets. It's exactly like to say that Christopher Columbus had begotten America. Everyone would laugh. Well, well, mathematics, it's the same. The mathematician is not going to father. He will discover and he will discover a new section of mathematics. Or because if you want, this math comes from physics. And of course, I think it was Hadamard who spoke about it the best, if you will : mathematics comes out of physics because they have to do with external reality, they have a particular taste. They have a particular force, but I don't know anything that generates, no.

2 Mathematical research

We are trying to understand. We try to understand physical reality, of course, and we try to understand mathematical reality. This is something that is very, very obscure, very difficult to understand. And the way we try to understand is to work out, so here we invent concepts. These concepts are very specific. For example, I talked about the concept of topos due to Grothendieck. These are very specific concepts.

These are not vague things, they are very, very precisely defined things. And what is extraordinary, if you will, is that one of the often overlooked roles of mathematics is that of generating concepts. And these concepts, at the start, are going to be purely mathematical concepts. But gradually, they will become part of everyday life that we all share. A very striking example, it's the concept of function, you know, the concept of function is not something that is obvious to the general public, etc.

But when we talk for example of the slowdown in unemployment growth or things like that, that corresponds to very precise mathematical properties, defined on functions. And so, we have there a striking example of a concept which comes from mathematics and which, gradually, gradually, will be part of the common baggage of civilization. And one of the reasons why he can register, is that now, we don't only have printing, writing, we also have computers. And the computer is not only going to be able to transmit words, to transmit numbers.

It will be able, precisely, to transmit functions, that is to say that we will be able to see on its computer screen the graph of a certain function, etc. And we will be able to understand qualitatively the properties of these functions and the relevance of mathematical concepts.

3 Between physical reality and pure mathematics

There is always a balance and precisely, my balance... if you want, we can only move forward if we walk on two feet. My balance is between physics, of course, that I never gave up, never, because there is this essence of quantum physics, precisely, which, as I said if you want, allows this coexistence of the continuous and the discrete which is magnificent and on the other hand, there is the algebraic geometry, number theory, etc.

I was talking for example about topos. Grothendieck wrote on the topos that precisely, it was "the bed at two places which allows the marriage between discrete and continuous". So although this is a very different approach, it is not totally disjoint.

So there is this balance between the two and hey, physics, of course, comes up against experimentation. The Mathematics also collides, in a way, with experimentation. I use a lot the computer, I use a lot of computer checks, even for things that seem impossible to watch on the computer. And there, we come up against a real reality. We run into something we can't change. We want to know if something is true or not, we do tests, we look at this.

Well, well, it's a bit like a physicist who will do experiments and see if his idea is correct or if he needs to correct it. Okay so there are these two sides of my job, if you will, and there is not one side that took precedence over the other, they have always remained very balanced.

Some people say that your noncommutative geometry is a bridge between quantum mechanics and classical physics. Why ?

Yes. If you will, what there is... it's not really a bridge between quantum mechanics and classical physics. No, the bridge between quantum mechanics and classical physics is what we call dequantification. It's a whole story, it relates to characteristic 1 which I was talking about earlier. But it is something else. In fact, noncommutative geometry, no, it's more a bridge between the quantic and the geometry and the fact that, precisely, our geometry, the one to which we are accustomed, the Descartes'one, applies perfectly to classical physics, but does not apply to quantum physics.

Quantum physics means rethinking geometry. This is exactly my job. It's exactly what I do, what we did with my collaborators, is to show that the Lagrangian [3], which is extremely complicated, and which contains both gravitation and quantum mechanics, the Lagrangian of quantum mechanics, this Lagrangian is understood in an incredibly simple and conceptual way when we have the tools of noncommutative geometry. But it's still a Lagrangian you have to understand, which is at the classical level, that is to say that it is not yet quantified. So we know we're on the good track because this Lagrangian who looks incredibly complicated, it takes four hours for putting it into formulas...

A Lagrangian, can we summarize it as a formula ?

A Lagrangian is a formula, but basically, if I want to explain to you what a Lagrangian is, we must understand the simplest principle of action, which is what is called the Fermat's principle, and I can explain it to you in three words. Fermat's principle is the principle that says that light will follow the shortest path for it. So you can do an analogy with... Suppose you are in the suburbs, and that you want to go inside Paris, for example.

Ok, well, then, if you know there are big big traffic jams in Paris, what you will do is that you will reach the point of the circumference of Paris that will be closest from the point you want to reach ? It doesn't matter that you're not going in a straight line. And that is what is called the principle of refraction of light. Okay ? So the Fermat's principle tells you there is a principle of minimizing travel time, ok ? So the physicists have enlarged this principle to much more general things. They enlarged it to all physics and when it was enlarged says to all physics, the principle of action is precisely what is called the Lagrangian. Okay ? And it contains all of physics because it contains both gravitation and it also contains the Lagrangian of quantum mechanics. But we are still at the classical level, as one says. One needs also to quantify all that.

And so, what we said with my collaborators is that quantifying something that we don't understand, it's a bit illusory. And so, what we did was that we understood this classic Lagrangian, as being the Lagrangian of Einstein, that is to say pure gravitation, but on a space a little bit more subtle than space, which is simply the continuum we are used to and the space we have found, it is a space which is pre-

3. The Lagrangian of a dynamic system is a function of the dynamic variables which makes it possible to write in a way concise the equations of motion of the system in quantum mechanics. But we are still at the classical level, as we say. We still have to quantify that.

cisely a mixture between the continuous and the discrete, and this mixture can occur only through the noncommutative.

And what does this give you, to have found all that ?

It gives us first a great aesthetic pleasure, the fact that a Lagrangian, who normally takes four hours to be transcribed into LaTex on a file, can be written as a very small formula. And this tiny formula is even simpler than Einstein's formula, since it's a formula which only counts the number of eigenvalues of the length element in noncommutative geometry, which are greater than a given length. So, this is something incredibly simple and that's something if you like, which rightly says that the length element in non commutative geometry is something totally different from the conventional length element. The element of classic length, you know, it was the standard meter that we were told about when we were students and we told ourselves "The length element... The standard meter is deposited at the Pavillon de Breteuil", etc. And there was a whole story that explained the creation of this standard meter with Delambre and Méchain who had been sent, the surveyors who had been sent between Dunkirk and Barcelona to measure, etc.

So what ? An extraordinary episode happened in the 1920s, which is exactly the same in physics as the paradigm shift we are proposing for noncommutative geometry. This episode is the following. There was a congress, not of surveyors, but of the members of metric system. So people were gathered and among them, there was one who raised his finger during the meeting and he said "I'm sorry to tell you some bad news, but the unit of length changes from length.". Imagine, the meter changes length. So it's very, very annoying, if you want, we have a unit of length that changes in length.", then the others asked him. "Okay, well, it's very well, but how did you know that ?". He says "Look, I took the yardstick which is at the Pavillon de Breteuil, etc. And I measured it against the wavelength of the krypton and I realized that it has changed length".

So, disaster, etc. We cannot take an element of length that changes length, then gradually the physicists reflected, etc.

And they understood that in fact, it was necessary to take as a unit of length, which had made it possible to see that the old unit of length had changed. So they took a unit of length which is spectral. Then they replaced krypton by cesium.

And it is quite obvious that if we want to unify the metric system, let's admit in the galaxy, it will be necessary to give something convincing.

If we tell people "if you want to measure your bed. You must come to the Pavillon de Breteuil, etc." Well, it won't be very convincing. If, on the other hand, they are told "listen, you are taking hydrogen. You take the spectrum of hydrogen. There is a certain spectral line which has a certain form and you take its wavelength as the unit of length". It's amazing. Well the change that allows to go from classical geometry to noncommutative geometry is exactly the same, i.e. that in noncommutative geometry, the element of length is spectral, it is given by the inverse of what is called the Dirac operator and it is given by what physicists call the propagator of fermions, that is, something they always write as an infinitesimal. So there is there, if you want, a coincidence which is extremely, extremely strong, and which says that there is an evolution of geometry which goes from an entirely classical formality, entirely commutative to a formalism which frames with the noncommutative, but which is also spectral, which becomes spectral.

Why do you say you've not finished... ?

We're not done, but no. We are at the very beginning, if you like, we are at the very beginning. First because well, actually, we would have to go to the quantum level for the geometry of space-time, i.e. quantify this Lagrangian I was talking about, but also in understanding, for example, the geometry which underlies the prime numbers, we are still far from the count. We recently found therefore, the object we have been looking for fifteen years.

That's what I said. It is an object of algebraic geometry, but which uses very sophisticated concepts. because it uses both topos and characteristic 1.

But on the other hand, when we give the definition, the definition is overwhelmingly simple, if you want, therefore, it is surely the right definition. But then you have to develop the analog of the algebraic geometry which had been developed in finite characteristic, it must be developed in characteristic 1, it is necessary to develop a cohomology which replaces Weil's cohomology, etc. So you see, there is a whole program there, which is in front of us.

4 The tools of the mathematician

We have an incredible chance in mathematics, it is that a mathematician faced with a very difficult problem, what does he do ? In general, the problem is too difficult to attack it head on. So there is a method. You should know, for example, that if I tell you "we take a bar of chocolate that has 4 on one side, 8 on the other. How many times does it take to cut it in half so that it is finally reduced in small tiles ?". You'll tell me it's very, very complicated, etc., okay.

Well, the mathematician's idea is immediately to generalize the problem. That is to say that instead of saying a chocolate bar of 4 times 8, he will say a chocolate bar of n times m, where n and m are two integers. And then after, he will take the smallest values of n and m. For example, it will take $n = 1$, $m = 2$. It will take 2 tiles. Ok, we cut in one go, it works. Okay. And then after, he will have fun looking at simpler, but more and more complicated cases. And after a while, because he will have solved the simplest cases which are easy, the difficulty will increase like a staircase. And through this staircase, at a given moment, he will say "Ah here, that's it, I understand !" and he will have understood the general idea. Okay.

So that's the essence of mathematics. And if you want, there's a great thing, that in general, when we look at the small cases, the simpler cases, well, then, we will be able to proceed by analogy.

And analogy is a tool of mathematicians who, for the moment, completely escapes the computer because the analogy is never exact. Analogy is something...

Is it intuition ?

No, it's not intuition. Intuition is something else. Analogy is something that consists in saying that we are going to transplant methods that have worked in one case, in another case. And of course, that will not be exactly the same. You'll have to take... it's like taking a small flower, you transplant it elsewhere, if you want, it must remain alive, but the earth will be different, it will be in a different context, etc.

This idea of transplanting it, is that intuition, saying to yourself "well, I'm going to take that tool, and I'm going to bring it there" ?

Yes, well, it's true, if you want in mathematics, there is a significant part of intuition which is very, very difficult to define.

5 Intuition

It's true, it's perfectly true that in certain situations, certain situations where we have a very difficult problem, etc., you get to have an intuition. But this intuition, if someone asked you "Can you tell me,... there, what do you want ? What are you doing ?". Etc. We would be unable to say it, we would be unable of saying it, because it is an intuition which is not yet rationalized and which, if we tried artificially to rationalize it, would evaporate.

And that is something very, very difficult to understand. This is something that is very difficult to formalize and which makes the mathematician's work very difficult, it is that his entirely work, is purely rational, if you will...

Neither linear.

Neither linear, absolutely not linear, that is to say that there are periods, often quite long, in which there is a kind of incubation. Hadamard spoke about it perfectly, I will not repeat what he said, but there is an incubation period which is often long, and which precisely requires not to be too fast intellectually, because if we are too fast intellectually, we will easily find reasons why it is not going to work.

But it is often a mistake to believe that because you have to let things evolve slowly. It is necessary to be extremely patient, but at the same time you have to be exactly like a wild beast with watch out, that is to say while being patient, stay fully awake and be able, precisely, if you see something that seems to work, there, you have to jump. Of course, you shouldn't be asleep. It is not the good way to wait for it to fall from the sky like that.

At the moment, what is the beast you are tracking ?

Well, the beast we're hunting down there was what I told you, it's about number theory, etc. This happened with Katia Consani, precisely, it was that we had the impression that there were a certain number of pieces of the puzzle, this huge puzzle that underlies the prime numbers that have fallen into place. So hey, it's a breakthrough, it's a breakthrough, but it's a breakthrough that may seem too naive, etc., on a certain side, but for us who understand most of the elements that make up this puzzle, it was truly an eye opener.

Of course, we are very, very far from the goal. We are still very, very far from the goal, but that allows, if you want to hang on to the whole panoply of tools, concepts that have been developed by surveyors algebraists, so first André Weil, then Serre, then Grothendieck. And so, it gives us a species of work program. And that's great. That's great. You mean that ultimately I would say that it is more fun to have a work program, namely that we will embark in this work program, it is a bit like the sailor who embarks on long journeys, etc., there is more fun to this than to finish something, because it is open, and that, it opens something.

6 Mathematical reality

Many times you talk about mathematical reality, but explain it to us, because it is foreign to us

The problem, if you will, is that mathematics is not something you can understand, or that you learn by reading, without doing it. In this, mathematics are very different from other subjects. But mathematical reality is something incredibly concrete, it's as concrete as chair, if you want, that you can touch. But I will not try to give you examples of arithmetic, because it's too simple. But for example, let's take geometry, if you want, if you take ordinary Euclidean geometry, I can give you a statement. And then you can, after, seek to understand, seek to see if it is true or not. And I give you an example. That's what we calls Morley's theorem. It is a magnificent theorem. It is a theorem that tells you that every triangle generates an equilateral triangle. So how does this equilateral triangle emerge ? It emerges as follows : you take the triangle and you take each angle of the triangle and you cut it into three, three equal parts, okay ? So you get lines like that. You intersect these lines, it will give you three points. Well, these points are the three vertices of an equilateral triangle, regardless of the triangle you're talking about. So it's amazing, it's amazing. So you can tell me "but no, it's not true !". And me, I will give you the demonstration that it is true and we touch the mathematical reality.

So in fact, we also touch it in an extremely concrete way with computers, that is to say that well, we can... we can ask ourselves the question from when we convince ourselves that something is true, but one convinces one thing is true in two different ways. There is an experimental way. It's exactly like in physics, that is to say, well, there can be a statement, I don't know, on modular forms.

But the computer is so powerful, so strong, if you will, that it is capable, precisely, to calculate examples. And if we check on enough examples that it works, we are convinced that it's true. It is not at all the same thing as finding a demonstration. But you have to understand that it's a bit like physical reality and it's a reality that is there, that is tangible and that we can explore. We can explore it directly. We can explore it by thought, it's much better and you can also explore it through the computer. And this reality is there all the time. It is not, how to say, that you can touch itas you touch physical reality. But whatever, it is just as real. It is just as fundamental as that. I'm going back to the example of Morley's theorem, if you want, what will remove the doubt completely, is to give an algebraic demonstration. Such a demonstration exists. There is a purely algebraic demonstration of Morley's theorem.

And then, once we find this demonstration, it's great, that means that first, it works in any case, of course, because it's purely algebraic and besides, not only that it works in all cases, but the geometric figure we have, it will use what is called the field of complex numbers. But the algebraic demonstration will work for any field, so it's a great thing because it means that starting from a geometrical image and a geometric intuition, well, we found an algebraic formulation which is much more general. And so, we have taken a step, we have taken a step precisely by this kind of communication, if you want, between on one side a geometric intuition and on the other side, an algebraic intuition, which is an algebraic formulation of things and that is much more powerful in a way. But there is always a round trip, that is to say that some mathematicians have a geometric view of things, have images, mental images, others have an algebraic understanding of things, that is to say manipulation over time, unlike geometric manipulation, geometric understanding.

7 noncommutative geometry generates its own time... (?)

I would like to dig a sentence that I did not understand : "geometry generates its time".

Basically, the way time appears is that in geometry, when things don't switch, when ab is different from ba, okay, but there is an equation that occurs that I'm going to over-simplify, of course, which is that ab is not equal to ba, but is equal to b, multiplied by a transformed by time, but not by the real time we know, but by imaginary time. So the bottom line is that ab is not equal to ba, but is equal to b times a transformed by imaginary time and then, by what we call analytic extension, well, we come to a real time. So that's the basic mental idea, if you will. So at the start, it was an idea that was developed by the physicists, called Kubo-Martin-Schwinger (KMS), then by other physicists Hugenholtz-Winnink-Haag, and then by Tomita, a Japanese mathematician, and Takesaki.

And then I worked in my thesis with Jacques Dixmier on it and I did precisely, at a moment, a really fundamental find, which was that when we had the impression that this evolution over time depended on a particular choice, a state, I had found that it did not depend on it, by what we call "modulo the inner automorphisms". So it gave all a multitude of invariants of the algebras in question, and it allowed to classify them, it allowed to move forward considerably, but there was how to say... There was a philosophical message that I had felt through my intuition, of course, in this discovery and which I had for years been unable to link to physics, until the day when I met a physicist a little by chance. And he was Carlo Rovelli.

And talking to him, if you want, I realized that... we both realized that he had a similar idea, but he didn't have the mathematical tools to put his frame on feet. And in addition, he did it in what is called the semi-classical framework, that is to say, not still quantum. And so, putting it all together, well, we understood, if you will, that it's this equation, this generation of time by the noncommutative, that probably had a fundamental role in physics, role which is not yet fully established.

8 Make others admit the scope of a discovery

One of the pretexts, how to say, of a book that we have just written, which is called *Le théâtre quantique*, precisely with my thesis professor Jacques Dixmier and with my wife Danye Chéreau, definitely, if you want, to get this idea across to the general public, because well, finally, I hold very much on to this idea and get it across to a message that is fun, if you will, say a message that recounts the episodes in the life of a researcher, the heroin, etc.

And you would say that there, at this moment, we are at a turning point, precisely, on this concept of time.

You know, I don't know, because you have to understand that in the field in which I work if you want, there are two levels. There is a level which is the level of research, how things are going, how they are evolving. And then, there is a sociological level. And the sociological level, it is to what extent these things will pass in the scientific community. And obviously, these are two things that are disjoint, of course, okay. But hey, I cannot say that I have been very concerned during my career with the sociological level. So hey, then it's still a little behind...

You can pronounce yourself.

No, not really, no, let's say it's a little behind, it's true. Sometimes it's exasperating because that we see people who don't understand, etc. We see other theories that occupy the front of the scene a bit indecently, but that's how it is. So, I mean, we often did.

When we work, we have a choice. Either we really work, or we spread to say, or we communicate. Well, it's difficult, it's very, very difficult to do both things at the same time. This is almost contradictory. Why ? Because when you really work, it's an occupation of all moments and we have very, very little free time, actually. For the rest.

One works alone ?

No no no. We absolutely need others. I'm going to say, I've always had collaborators, either in physics, especially, for example, my collaborator Chamseddine and good, right now for number theory, Katia Consani. So, it is essential to have collaborators, precisely, because not only, they bring new ideas, of course, but above all also, I would say, not to be completely alone, because there are obviously very difficult times. There are obviously moments, long, long periods in which nothing happens. And if we were alone, we would have maybe more chances to get discouraged than if you work harder. That plays a very, very important role.

9 The intensity of the courses at the Collège de France

I have an audience, quite varied, very varied, who regularly comes to my classes and there is a very strong contact which is established with the public.

So it's really, with you, the research being done...

Ah yes ! And I must say that, how to say, the leitmotif of the Collège which makes its originality, that creates precisely during the months which are the course months at the Collège, an extraordinary period. Why ? Because good years don't happen all the time. The good years, basically, two weeks before the class, I know what I'm going to talk about. Basically, I have the subject, okay, but I have absolutely no details. And basically what I do is make sure I have two weeks in advance, that is to say that I know roughly what I will do in the next course. And then I work on the next lesson. So I'm looking for the next lesson and it's a period of absolutely incredibly intense research. Why ? Because we often underestimate the fact that in mathematics, if you want to understand things, you can feel like if you have understood. But there is always an extraordinary benefit to go into the smallest detail and, how to say, to exhibit all the facets of a concept, etc. Because...

You are perfectionist

Be a perfectionist, because it will generate things. And I saw thie in the course again this year because it is precisely through all the development of the course, etc., that the object of geometry algebraic appeared. So what I mean is that it's a very intense period, very hard, very hard, because it's physically hard, for example. I often have physical manifestations, if you want tension, during the lesson period, at the start, etc. But it's extremely productive because it's the period when I work the most, etc.

But it's concentrated, you can't do this all the time, obviously. If we did that all the time, we would be completely washed out, raplapla, etc.

Perhaps it is the secret of your profusion...

Yes, absolutely. Absolutely. Because if you want to often, I got myself. I thought to myself "well, maybe if I would have stayed at the CNRS", etc. But it depends. Of course, each mathematician is a special case and if you want, you have to find the equation that will allow him to work. But finally, in my case in particular, the motivation that there is in Collège which consists in saying that each course must be a course on something original that is being done, at the limit which is not published, but is really in the making. But this motivation, I find it great. I find it great. Years ago, indeed, when it was much more difficult or much more annoying, but that's how to say, it's very, very well dosed. That is to say that if you want, it's not as if I had 6 months of lessons, that would be too much But this course period which lasts between 2 and 3 months, if you want, it's perfect because it's very well dosed and it requires a certain discipline. It forces precisely to stay all the time, all the time a little worried, a little on the wire. And never, never say to yourself "Good, bah, ok, now I'm old, etc.". We can't think this, we can't. Because, well, of course, I will have to stop at some point. But we absolutely must not do that because if we do that, we will not be able to do the course.

10 Doubt

Doubt, you know, in the book, the book with Danye Chéreau and Jacques Dixmier, begins with this exhibition in Venice which is called *The Praise of Doubt*. Doubt is present all the time. Doubt is present at four in the morning, when we wake up at night and say to ourselves "Couldn't I have make one error there, etc.". And let's start checking. So the doubt is present all the time, but I will add nevertheless that from time to time, from time to time, it seldom happens, but it happens, fortunately, there are times when doubt evaporates. And precisely, I doubted for years and years that the space I had found in 1996 for prime numbers was the right one. And what we found recently with Katia Consani dispels this doubt. It is surely the right space, therefore, it is wonderful that from time to time, there is, if you see like that a way to remove the doubt. So of course we can say, "Okay, well, you have raised the doubt for yourself, raise it for the others.".

Like the goal of all these developments, if you will, it's very, very difficult. Of course, this is not immediate, but for us it is very, very important to remove the doubt, even punctually, if you want, like that, on such an important notion.