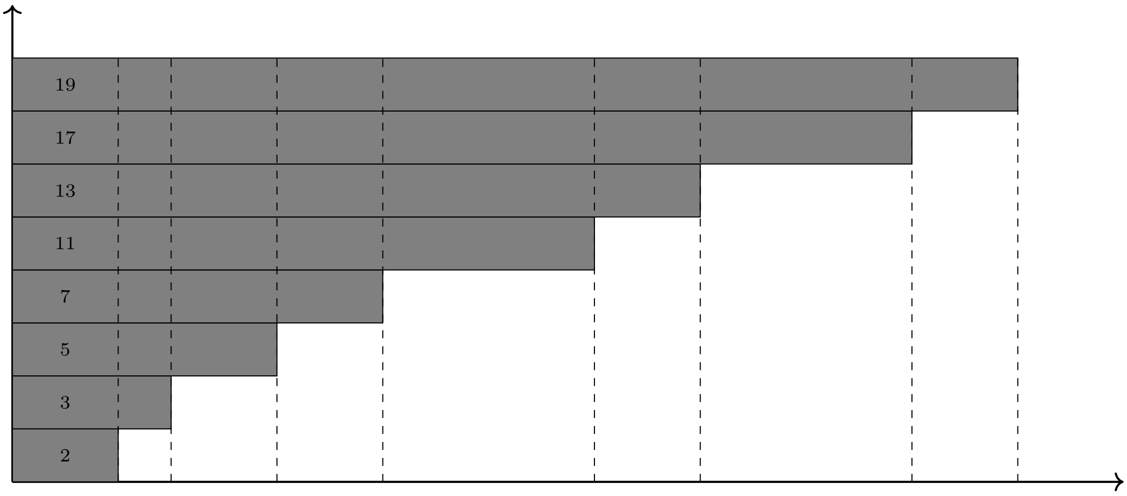
Primes quantification (Denise Vella-Chemla, 28.8.2018)

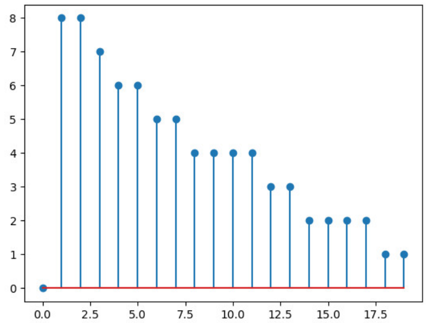
Recently we discover a way to quantify Goldbach'decomponents of even numbers (i.e. they are the primes intervening in even integers'decompositions in sum of two primes), that is to say a way of counting rollups of those numbers by cutting them into pieces.

Here, we wish to study in the same way primes themselves.

So let us stack them on each other :

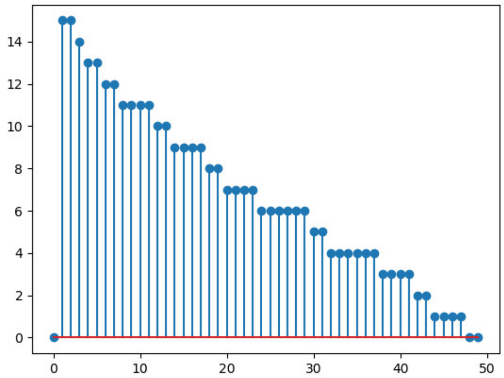


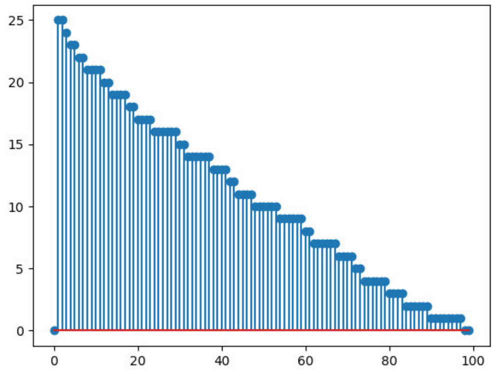
Let us make quanta go down as much as possible (as if they were attracted by a sort of gravity !) and let us count those rollups, we obtain the following diagram using a python program :

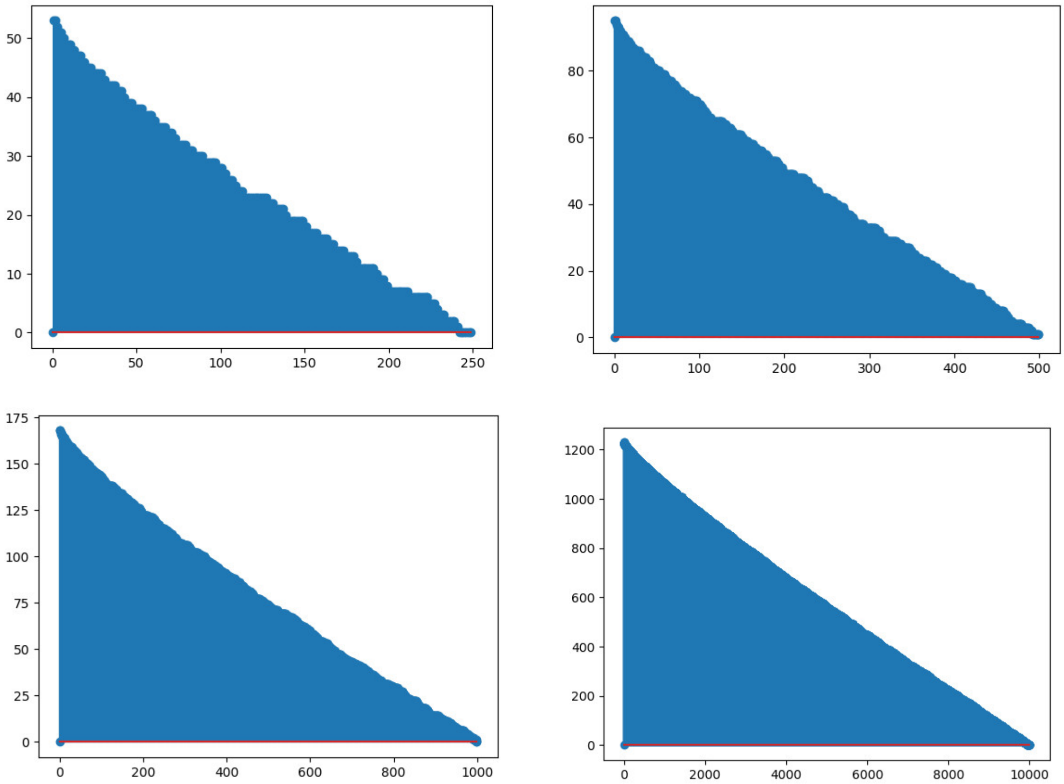


It is the diagram of a function, $f_{20}(x)$ that provides for each $x \leq 20$ the number of primes that are between x and 20. Making this, the abscissa at the origin is equal to $\pi(20)$ (with $\pi(x)$ the usual notation for the number of primes that are smaller than or equal to x).

Below we provide discrete curves corresponding to similar countings until 50, 100, 250, 500, 1000 and 10000.







As expected, were obtained diagrams of decreasing nearly linear functions $f_n(x)$, that seem to have as equations $f(x) = -\dfrac{\pi(n)}{n}x + \pi(n)$ with n the greatest abscissa, while they are in place of the form $f(x) = \pi(n) - \pi(x)$.

One can stretch $\pi(x)$ as following : for image values to cover the whole interval $[0, 20]$ for instance, when a prime number is encountered, vertical steps are cumulated in such a way that steps are of height $p_k - p_{k-1}$ rather than being of height 1 (like for $\pi(x)$ function), this permits to stay near the diagonal of length $\sqrt{2}n$.

The corresponding function consists in associating to x as image the greatest prime number that is smaller than or equal to x. In this way, prime numbers are on the diagonal of the diagram.

This idea is illustrated below presenting *stretched* $\pi(x)$ for x smaller than 19 (on the right) regarding $\pi(x)$ (on the left).