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And the others ? Stories and imaginaries Catherine Goldstein

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Well, hello, thank you for this invitation. So I have to introduce myself, I think, so my name is Catherine Goldstein, it's on the board, I'm Director of Research at the CNRS. It's already been said, I deal more with History of Maths but I'll come back to it.

But I work in a Mathematics Laboratory, so the Mathematical Institute of Jussieu Paris-Rive-gauche and I depend as Research Director at the CNRS, I depend on the INSMI, that is to say the CNRS Maths Institute, not the Institute of Human and Social Sciences. So let's say that among the mathematicians who are going to speak today, I am the bat so if they are mammals, I am the bat a little bit, that is to say I I started as a number theorist at Orsay, I was hired at the CNRS as a number theorist, and if I became a historian it is precisely relevant, let's say, for what I am going to tell today, that is -to say that I did not come from an academic background at all and I was struck from the start practically by what I really experienced as a contract, even a contradiction, between a very strong feeling emanating from my colleagues, of a community of mathematicians and mathematicians, and I think that was quite clear in Isabelle's presentation at the beginning: it is exactly this kind of impression, that is to say that there is a close-knit community, quite large, which has fluid collective organizations, also very important collective values yours, integrity, concern for truth, proof, etc., rigor including in administration, attention to others at least within this community therefore a strong sense of community; and then at the same time a presentation of self particularly to the outside that was individualistic, very elitist and I would say really based on a success through the centuries of white western men who looked like fully manage the evolution of the domain.

And for me, it was totally bizarre and contradictory, and it was this contradiction that I tried to resolve, by putting myself to think about it in several ways, I will talk about it, and finally, inside a new research discipline which was the History of Mathematics, which seemed a reasonable discipline to fix at least my professional research activity, and at the same time to continue to reflect on these questions. So that's what I would like to talk about today, that is to say this kind of contradiction and what I can understand in a certain way; it will obviously be linked to the other two presentations. To show you a bit in detail what I mean by this, I wanted to show you immediately read to you immediately a presentation that was made by the President of the Société Mathématique de France in 2009 so it was Stéphane Jaffard; it could be done now you are going to find themes that we have already heard this morning, uh, earlier; it could have been done twenty years before too, that is to say that it is really a discourse which in my opinion is very characteristic, very typical, of what we hear within the community in particular of maths pure. So I read it to you briefly: “Mathematicians have long felt the need to make

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their discipline which is unloved. Doing so has now become essential following recent changes in the management of research, the main orientations of which are increasingly set by politicians or decision-makers […].

Our discipline feels more directly called into question than the others by these upheavals [...].

The network of mathematics laboratories patiently woven over the past 20 years *(so that's what Isabelle alluded to earlier, incidentally)* has enabled mathematics to maintain itself at the highest level in the world. This network is now threatened by a policy that risks encouraging the withdrawal of research into a few so-called “excellence” establishments” (2009)

At a time when everyone is asked to prove their immediate profitability, when economics governs society, it has become difficult to justify maintaining free, non-finalized research, the ultimate goal of which is to extend human knowledge. We can at least argue that the most abstract and disinterested research often turns out, a few years later, to be a totally unexpected key tool in a great technological breakthrough...

Why does society have an erroneous perception of mathematics, even though these researchers feel that they are players in a booming field that is more than ever indispensable to other sciences and to technology?

In my opinion, one of the keys to the answer is in the hands of the media; it is almost impossible for us to explain the stakes *(we could say the same for other disciplines, moreover)* to explain the issues that lie behind the internal problems of mathematics, for lack of being able to highlight the spectacular practical achievements, including the interest is directly perceptible.

So I will quickly resume this speech because it is exactly the themes that we see in general.
So an unloved discipline, poorly known and a discipline at risk of losing a certain number of its criteria and its means of existence.

A community that is active, that is organized, that is extended.

Mathematics is useful to society, the basis of technological, scientific and industrial advances.

And then the double problem, let's say, which is "how to justify the theoretical part, the important part of mathematics, the heart, let's say, of the profession, the theoretical part?" is always put forward, that is to say it is not useful now but if you are patient, in the long term, it will be useful.

So it's this long-term affair and in my opinion it's over this long term, I think we've already heard about it, in Isabelle's presentation: it's this long term which in my opinion poses a rather serious problem in relations with politicians, the media, and other disciplines.

And then, how to communicate with these media, these politicians, etc.?

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So why I gave this speech and these outlines is because in fact this speech is part of a series of speeches from the mathematical society, from the mathematical community as a whole, which have at least started, at least for me, in the 80s. So I think it started maybe before but so I couldn't tell you about it before, and it's the series called “Mathematics to come” in particular. So there were several events, and large-scale events in the mathematical community, at least from 1987. So that is directly linked to the presentation before because in 87, it happened acted to solve two problems of the community: to get out of the Bourbaki era and it was totally explicit; that is to say that the Bourbaki type, without applications, without logic, without a lot of mathematics, with the elitist and very masculine side had reigned over the mathematical community and it was a question of changing that. So that was one of the important aspects for this operation. And then, also, something that was quite a shock, in fact, which was the creation of another mathematical society, in this community which was supposed to be unique and tight-knit, which was the Société de Mathématiques Appliquées et Industrielles (SMAI), which was created, I believe, in 83, if someone can confirm the date, so just before this organization.

And then an age pyramid inherited from after the Second World War which was becoming catastrophic, therefore with an obvious aging of the population and no renewal.

So that was the issue of the first congress. And it is therefore a congress that brought together round tables, industrialists, at Polytechnique, discussing the problems of society, applications of mathematics. But also a survey in 50 high schools, on the image of mathematics. This is also where my dear colleagues discovered that the image of math among girls and the image of math among boys were not the same thing. And that was the first shock, I would say, of kind, of the community. But also a brochure on maths which was distributed to six thousand journalists and politicians. So a real brochure, not just mimeographed etc., but on glossy paper; you have an image here which, well, was more or less taken up in the various “Maths to come” conferences.

But in any case who was already in action, I think this image is very interesting, compared to what it puts forward; it has been discussed a lot, so I don't have to comment on the iconography but it would also deserve that.

So that was a first operation, and then there were plenty of others: there was the year of Unesco: Maths 2000, it was the year of maths at Unesco. And then a second symposium in 2009, on the same basis as the previous one, but this time with other local issues, well, you know them: the autonomy of universities, the famous financial crisis which called into question mathematicians and mathematicians and mathematics and that we had to try to discuss, and a fall in the number of students this time less in the number of colleagues than in the number of science students, which particularly affected mathematics, and which had an impact on iron provision of comprehensive training, particularly in the provinces. So there were other local issues, but as you saw Stéphane Jaffard's speech, there was also this speech, which has been more or less the same for thirty or forty years now, from which we take up the essential points, what whatever the more local issues that lead to these operations.

So what I wanted to show you in particular is how these problems that are raised in these terms, find solutions which in my opinion create other problems and well, that's a little bit where I'm coming from. now.

And for that, I just wanted to mention the famous brochure distributed to political leaders

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and to the journalists of 1989. So I didn't have it on me so I'm sorry, but I would just like to explain to you how it was made. So there was first a double page of mathematical applications, to show that mathematics was part of everyone's daily life, therefore, with bank cards, planes, things like that, therefore illustrated by objects everyday or more technical techniques and technical drawings. And then, it was therefore necessary to illustrate the famous long term of mathematics to justify theoretical mathematics.

And there, the way it was done, it was essentially to put portraits, individual, of mathematicians, to try to justify the theoretical part of the math. Then, a double page on teaching; and then the problem of the age pyramid... What to do? What to want? etc
So this double page of theoretical mathematics, illustrated by portraits, I think is a classic in fact. I think that in a lot of cases when it comes to talking about mathematics, to politicians, or to a more general public, or being in contact with journalists, you end up talking about specific individuals, possibly, collective individuals like Bourbaki, but in any case centered around a few separate personalities. So, I'm going to show you my extraordinary, marvelous action in this affair: there were four of us to make this brochure at the end, and the first version, that was it, it was the portraits of Pythagoras, Pascal, d'Alembert , Poincaré and Bourbaki. And my intervention was to replace the bust of Pythagoras with a page from the elements of Euclid, with the Pythagorean theorem, in a medieval version in Arabic, okay?

To introduce Sophie Germain into the series of portraits and to replace Bourbaki with the photo of a contemporary mixed colloquium.

It was a triumph whose scope you will appreciate, I hope, well so it actually looks...well, I'm kidding a bit with that, but it was *difficult ;* there was absolutely nothing obvious about it. First then, with all the arguments you can imagine, i.e. Pythagoras, it's still the only theorem everyone knows, so we can't miss it, even if the idea that mathematics was born in Greece, made a great leap, and found itself in France in the seventeenth century, it is nevertheless an idea that has been completely swept away by historiography for decades. But... poof! Chase the natural... So we needed Pythagoras, well, so my battle was to show how he had really circulated, this famous Pythagoras. To introduce a woman who was not at the mathematical level of Poincaré etc., I was also told, well, that, I don't think we would say that anymore, but I was also told that maybe you needed a more attractive woman than Sophie Germain, because otherwise it wouldn't attract the students. I showed them the portrait of Poincaré again, good, but I mean, it's amusing, but it's at the same time terribly instructive, I think, of what's at stake.

So we have History as a tacit but omnipresent element of the interface between mathematics and society. This is justified by the fact that it is really a question of showing the long term of the mathematical results; this long-term impression, it is very strong, very important, I think, for this community, it also creates this community. The problem is that obviously, how do we tell this story over ten thousand years, well, we tell it with little bits of characters and that introduces other effects. Well it's often a story that is produced by the mathematicians themselves and it must be remembered that Bourbaki created, also wrote a History of Mathematics, which is also completely in the spirit of the Bourbaki of a certain way, but they also created this. And Weil was also a historian.

Well a story obviously reduced to relatively isolated heroes, and built around long-term identities, identities of proofs and theorems etc. And we find the same thing in the

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presentations of math, for example, to first-year students; I saw that again recently in an L1 auditorium, where it was explained that the Pythagorean theorem, or another theorem, is a theorem that hasn't changed for thousands of years.

So as I'm writing on the board, actually, elitism, which got kicked out, let's say, out the door, really, and we're trying to get it out and well so I think we heard Isabelle on that , in fact, enters through this window of the pseudo-historical presentation of mathematics when one is in an interface with an audience, then of children, students, journalists or politicians.

And for example when they reintroduced History of Mathematics in the Capes curriculum, well, it started with “it is important that secondary school teachers know the biography of key characters.” And the essential characters were Pythagoras, Thales, blablabla, etc.

I pass you the biographies of Pythagoras and Thales. So, therefore, faced with that, as I said, we have nevertheless tried to work among historians of mathematics and science for decades, to transform this image and study things differently. So, and it's quite striking that the historiography of mathematics over the past thirty years has been desperately trying to challenge these images in so many different ways: so either by studying known, established, say, structures of the community mathematicians through the centuries, so it could simply be the history of the Société Mathématique de France, the history of certain Academies where there was collective work that was done in various ways, long before Bourbaki, the history of scientific journals, mathematical journals in particular, how were they made, how were evaluations made, how authors were chosen, not chosen, things like that, or straightforwardly, by reconstructing networks, collectives, circulations of information, circulations of theorems, around people, texts, concepts, etc.

So I'm not going to go into that because I don't think that was the purpose of the operation, but I just put some works that go in this direction on the board, but finally my slide should be covered with texts, articles, etc. which all go in the same direction. So I wanted to give two examples, but maybe I'll only have time to give one, no doubt.

So an example borrowed from Caroline Ehrhardt is the example of Galois because it is an example centered around one of the heroes of mathematics clearly. So the standard story of Évariste Galois, I think there are a lot of people who know it even beyond the circle of mathematicians and mathematicians. So precocious genius, beginning of the 19th century, mistreated by the institution, since it was refused at Polytechnique; his brilliant manuscript is refused at the Academy of Sciences; it is then lost, in part, so we really have "the total", because this manuscript and its works already offered a vision which is often qualified as pre-structuralist, so he more or less saw, invented the theory of groups before it exists. A Galois who is also politically committed and on the right side: he is on the side of the revolutionaries when it has to be, who dies in a duel while writing a letter-testament of mathematics, therefore truly a romantic hero and remained misunderstood for decades, precisely before it was recovered by Galois theory.

So what Caroline Ehrhardt and then others too, but hey, she wrote two books about him, so she in particular, showed, is on the one hand that these famous successive refusals to the Academy

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etc., it was part of the fairly commonplace standards at the time, that is to say that many manuscripts are refused the first time, because it is a question of standardizing the activity of young people who offer work of research. So there is a whole indirect teaching activity for young budding math ematicians, on the part of academicians, except that it turns out that Galois had a very bad temper so he took it more badly than other people but ok, other than that. So lost manuscripts are quite common too, Hermite, for example, who was much more reasonable, also saw his manuscripts lost and a lot of them have been lost. If we start to look in detail at the timetable of the academicians of the time, and the number of reports they made, it is absolutely delusional. Well I don't mind that they weren't raising their children but it was still quite impressive, the number of reports and the extent of the reports that you claimed but earlier, the extent of the reports that are done, which is to say that sometimes they found better solutions. There is a whole process of proofreading the manuscripts here.

In fact the results of Galois were diffused rather quickly, they were also integrated, but not in the good point of view, that is to say that they were integrated by people who had other priorities than which will later be seen as the important priority, the essential priority of Galois theory.

And then above all there are these new, rather mythological stories that are set up about Galois as a misunderstood genius at the end of the 19th century and this goes hand in hand with a new position of the École Normale Supérieure, therefore at the beginning of the century, when Galois works, it is the Polytechnic School which trains the majority of mathematicians in France, the École Normale Supérieure is a more mediocre school, let's say, intended to produce teachers rather and that obviously changes during the 19th century, in particular at the end of the 19th century and Galois appears as one of the potential heroes illustrating the new situation of, say, the Grandes Ecoles in France and in particular the importance of the École Normale Supérieure. So there are plenty of collective phenomena like that, which we can put in place around the rather mythologized image of Galois which therefore shows another perspective on what happened around his work and himself too .

The other example I wanted to give was precisely the example of a discipline because that too is, I think, quite significant so we have myths around people, but we also have myths around the way in which history is made and mathematics evolve and in particular therefore, I returned several times to the question of the history of number theory, well, which was mentioned as, precisely, one of the fields of purest excellence and very well seen by the Bourbaki.

And so in the 1980s and 1990s, the history of number theory was a history that was reduced to, as André Weil himself says in the quotation on the board, “a small chosen group of men: thus the names of Gauss, Jacobi, Dirichlet, Kummer, Hermite, Eisenstein, Kronecker, Dedekind, Minkowski, Hilbert, come to mind. So a number of people like that and then a story that was centered around a central type of outcome, that acted as a driver of all development, so in this case it's called the law of reciprocity, well I I may not have time to say what it is, but it is a law that brings together the bricks of number theory, which are prime numbers. And then the extension of arithmetic, from ordinary integers to more complicated integers where the usual laws of arithmetic are no longer valid and therefore where we must develop new points of view to succeed in doing the same thing, in particular having a good notion of primes, which no longer works at the higher level of numbers, and which forces, if I may say so, people like Kummer or Dedekind to

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move to a more structural point of view, good.

And that therefore implies a single positive and fruitful dynamic, the establishment of this structural point of view, with purification of the methods, it is more or less the image that we have, moreover, of Bourbaki, I think , that is to say we purify the methods, like that. So we have an image, so I've made it a bit caricatural for you here, but quite linear with little heads talking to each other like that. Well, here it is, and above all the development of a piece of number theory, let's say, which we call algebraic number theory, from one great man to another, with a purification, an autonomization of the domain, and purification of methods. So I don't think I have time to give you the avatars of this in detail, I had planned to do it, but I think you're going to escape the math, we can come back to it later.

So at the end of the 20th century, out of curiosity, I went to see who was doing number theory, at the end of the 19th century, with this story in mind. And so I fell while stripping the journals on the diagram you see at the top so I don't know if we can see it well, but so in particular on the right, you see the distribution of articles written by authors of different nationalities and in black it's the French, and in light gray it's the German authors, and then where there are English authors with stripes, etc., etc.

* And that for me was really a shock because it showed hundreds of authors there are hundreds of articles involved, hundreds of authors and of all nationalities, whereas normally I should have had a majority of Germans and perhaps a little French lost according to the traditional story.

So vis-a-vis that obviously, one began to lean a little more in detail on what it was the evolution of the field during this time, including from the beginning of the century; we realized that there had been a lot of work done with developments in lots of different directions, not at all in just one, with overlaps in other themes, not only things like geometry for example which was not really expected, but also the probabilities, just to give examples on this and above all priorities let's say of an epistemological nature, which were very varied: not only purity, for example, there are really authors who see in the unity of maths a much more important spring than the purity of the methods and therefore they are very happy when you can put analysis, geometry, probabilities etc., with number theory, which is obviously completely anathema for a certain number of other groups of mathematicians who absolutely want to purify methods, to have methods that are purely arithmetic for example, or purely within e the theory of numbers, or else purely geometric in certain cases etc.

So we have tensions within a much larger community on priorities, the types of evidence that are acceptable or not, the types of developments, the fact of having applications or not, there are applications from number theory to weaving for example, including industrial weaving, in the North of France, at the end of the nineteenth century, and then the structuring of research, and that is something that occupies us a lot and has many busy ones, which are not always disciplinary, that is to say it is not always with a good manual that will fix the right concepts, and the right way of working with proofs, for example, of a certain way, or by trying to find the most general concept from which we can deduce all the results, as was described earlier. So there are in fact many other possible structurings, it can be structurings of letters, structurings, at a given moment we took over from Bourdieu the notion of field of research, with exchanges etc., but no agreement

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on what should be done, but a lot of exchanges, where people take up with their own priorities the same problems, the same statement, for example, and they will develop it in four, five, six different directions, and re-talk about it and possibly argue about whether there are directions that are better or not.

So that gives something, so, I'm not very good at drawing so I didn't manage to illustrate this really well, but just to show the impression of clutter much bigger than the pretty line I showed you see you later. And in general, then, not only are these epistemological priorities multiple, relations with society in general are extremely varied, even in the nineteenth century, but also, what seemed interesting to us, and it was also the example of Galois just now, is that *to consider some,* let's say, that is to say the great heroes of maths like *others,* that is to say to reintegrate them into multiple collectives, multiple networks in made it possible to understand much better even their practices, including their individual practices. So I think we also gain from re-understanding more collectively, in a more general way, including the famous great authors, who therefore need to be reassessed.

So, to conclude, this problem of the structuring of collective narratives, for me, joins a problem which is that of the structuring of the collective memories of mathematicians, because if I think many historians-historians of mathematics now would be agreement with the fact of looking more collectively, more socially, let's say the History of mathematics, changing the nature of these stories, changing the memory of mathematicians on the other hand turned out to be much more difficult. And personally, we were talking about responsibility earlier, I thought it was my responsibility, in fact our responsibility, to do so, and it's a patent failure, which I'm communicating to you: it's that is to say, we saw again last week a dispute about a presentation for a general public of mathematics, where we saw, again, the great men arrive, with the little anecdotes etc., that is to say say the same thing as described before.

I don't know, and I think it's open to discussion, what these difficulties are made up of, that is, there is the point that I raised at the beginning, that is to say the desire to have a long term of mathematics, which is not easy to transmit, but I think that there are also strong identification effects at the time of individual work, with a certain number of these famous characters, so this question of the difficulty of changing the random access memory of mathematicians in relation to their own past or their own organization or their own restructuring, I think that poses a problem and I don't know how to solve this problem, I still think it would be useful and important, also for the contemporary policy of mathematics to succeed in changing these images. That's it, and I'll stop there.